Chapter 1

Presentation of Data

Q. 1. Define 'Statistics'.

Ans. Webster defined statistics as "the classical facts representing the conditions of the people in a State, especially those facts which can be stated in numbers or in tables of numbers or in any tabular or classified arrangement."

Q. 2. Define the following terms:

a. **Class limits**
b. **Class intervals**
c. **Types of class intervals (Dec. 98)**
d. **Class frequency**
e. **Class mid-point or class mark**
f. **Relative Frequency**
g. **Pie Chart**

(a) Class limits
Ans. The class limits are the lowest and the highest values that can be included in the class. For example, take the class (10-20). The lowest value of the class is 10 and the highest value is 20.

(b) Class intervals
Ans. The difference between the Upper and Lower limits of a class is known as class interval of that class. For example, in the class (10-20), the class interval is 10 (i.e., 20 minus 10).

(c) Types of class intervals
Ans. Following are the types of class intervals:
(i) Where both the class limits are specifically stated either in an "inclusive manner or in an exclusive manner". For example,

Inclusive Manner		**Exclusive Manner**	
10-14 15-19 20-24	Both values included	10-15 15-20 20-25	Upper values excluded

(ii) The intervals may be represented by mid-points of the class intervals. For example,

Class	
	25
	27
	32

(d) Class frequency

Ans. The number of observations corresponding to a particular class is known as the frequency of that class or the class frequency. In the following illustration, the frequency of the class (10-20) is 3 and (20-30) is 5.

Marks	**Class frequency**
10-20	3
20-30	5

(e) Class mid-point or class mark

Ans. It is the value lying half way between the lower and upper class limits of a class-interval. Mid-point of a class is calculated as follows:

$$\textbf{Mid-point of a class} = \frac{\textbf{Upper limit of the class + Lower limit of the class}}{\textbf{2}}$$

(f) Relative Frequency

Ans. The relative frequency of a class is defined as the frequency of that class divided by the total number of measurements (the total frequency).

Relative frequency = (fi/n) for class i

Where:

fi = frequency of class i where i represents any of the classes

n = the total number of measurements

(g) Pie Chart

Ans. Pie Chart is used to show the break up of a total into component parts. It is used to represent the division of a sum of money into its components.

Histogram

Q. 3. A stock broker keeps a daily list of the number of transactions where his customers buy the stocks on margin.The following number of stocks were brought on margin by the

customers in January 2000:
(June 2000)

50	42	44	42	43	50
35	37	43	37	25	15
25	05	09	13	17	19
18	20	40	29	21	26
02	17	48	36	12	28

Construct a frequency distribution for the preceding data and then draw a histogram with number of classes = 10.

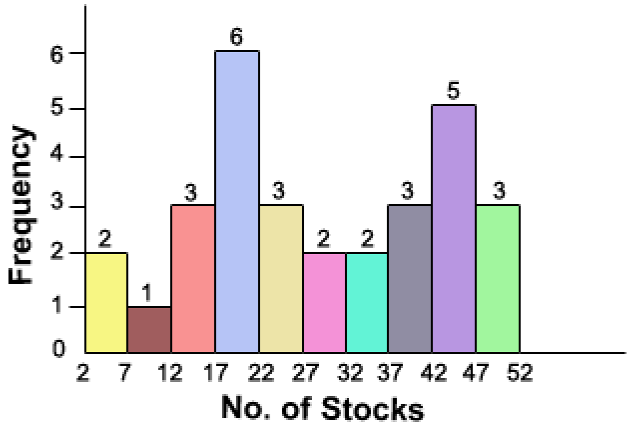
Solution. Given, the number of class is 10.
Highest Value = 50
Lowest Value = 2
Class size = Range/No. of classes

Where:
Range = Highest Value - Lowest Value
Range = 50 - 2 = 48

Therefore, class size = 48/10 = 4.8 or 5

No. of stocks	**Class frequency**	**Tally**
2-7	2	||
7-12	1	|
12-17	3	|||
17-22	6	~~||||~~ |
22-27	3	|||
27-32	2	||
32-37	2	||
37-42	3	|||
42-47	5	~~||||~~
47-52	3	|||

The histogram for the above problem is shown in the following figure:



Pie Chart

Q. 4. Draw a Pie Chart of the monthly expenses of a hostler, whose expenses per month are as follows: (Dec. 2001)

Item	Amount in (Rs.)
Food	2000
Room Rent	1000
Transport	500
Books/Stationery	500
Maintenance	1000

Solution. Since the total expenditure is Rs. 5000 (2000 + 1000 + 500 + 500 + 1000) and there are 360 degrees in a circle, we have to convert the percentage in degrees.

Conversion to % (Relative Frequency)

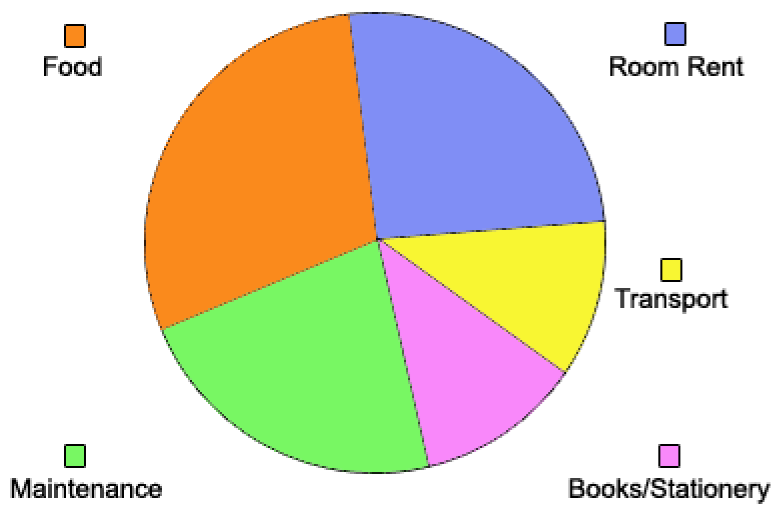
Item	Amount in (Rs.)	Relative Frequency (R.F.)
Food	2000	2000/5000 = 0.4
Room Rent	1000	1000/5000 = 0.2
Transport	500	500/5000 = 0.1
Books/Stationery	500	500/5000 = 0.1
Maintenance	1000	1000/5000 = 0.2

Conversion to degrees

Item	R.F.	Degrees
Food	0.4	0.4 X 360 = 144

Room Rent	0.2	0.2 X 360 = 72
Transport	0.1	0.1 X 360 = 36
Books/Stationery	0.1	0.1 X 360 = 36
Maintenance	0.2	0.2 X 360 = 72

The Pie Chart is depicted in the following figure:



Q. 5. The following table shows monthly living expenses for an MCA student of IGNOU: (Dec. 2002)

Item	**Amount in (Rs.)**
Food	100
Apartment	900
Transport	200
Entertainment	300
Maintenance	400
Miscellaneous	400
Total	**2300**

Draw a Pie Chart of the monthly expenses.

Solution. Since the total expenditure is Rs. 2300 (100 + 900 + 200 + 300 + 400 + 400) and there are 360 degrees in a circle, we have to convert the percentage in degrees.

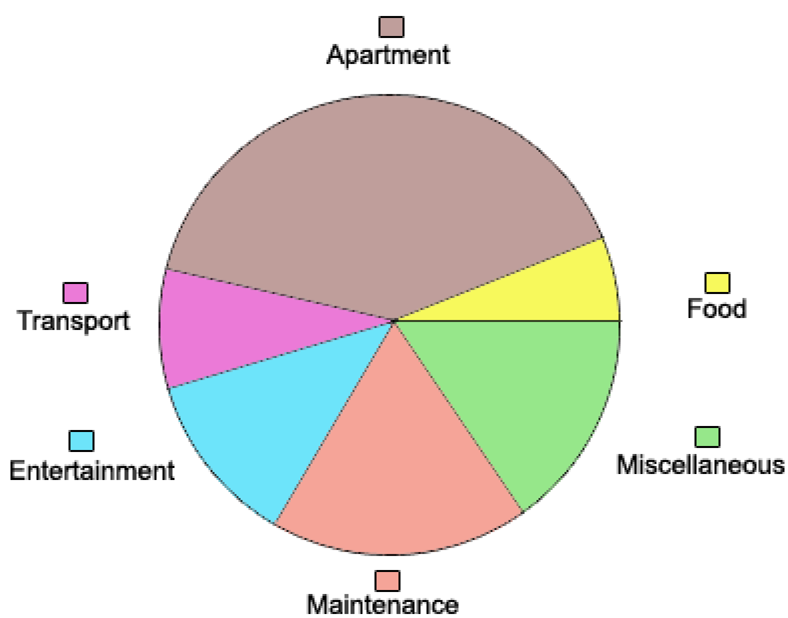
Conversion to % (Relative Frequency)

Item	Amount in (Rs.)	Relative Frequency (R.F.)
Food	100	100/2300 = 0.043
Apartment	900	900/2300 = 0.391
Transport	200	200/2300 = 0.086
Entertainment	300	300/2300 = 0.130
Maintenance	400	400/2300 = 0.173
Miscellaneous	400	400/2300 = 0.173

Conversion to degrees

Item	R.F.	Degrees
Food	0.043	0.043 X 360 = 15
Apartment	0.391	0.391 X 360 = 141
Transport	0.086	0.086 X 360 = 31
Entertainment	0.130	0.130 X 360 = 47
Maintenance	0.173	0.173 X 360 = 63
Miscellaneous	0.173	0.173 X 360 = 63

The Pie Chart is depicted in the following figure:



Ogive

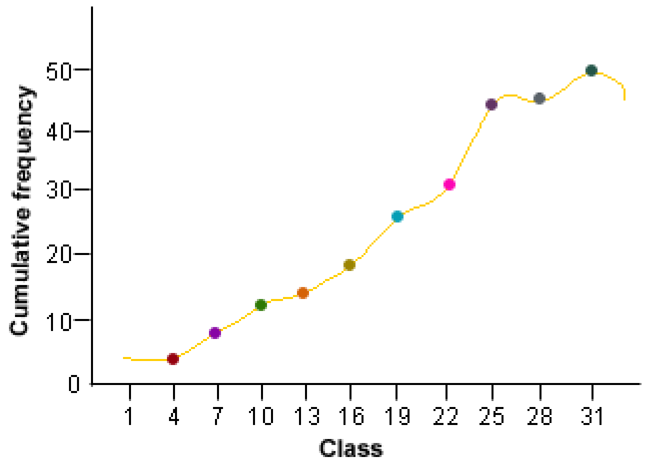
Q. 6. For the following class frequency distribution, draw the cumulative frequency polygon (or ogive) of less than type:(June 2001, Jan. 2001, Dec. 2000)

Class	Frequency
1-4	3
4-7	5
7-10	4
10-13	1
13-16	6
16-19	7
19-22	5
22-25	14
25-28	1
28-31	4

Solution.

Class	Frequency	Cumulative Frequency (<)
1-4	3	3
4-7	5	8
7-10	4	12
10-13	1	13
13-16	6	19
16-19	7	26
19-22	5	31
22-25	14	45
25-28	1	46
28-31	4	50

The following figure depicts Ogive of less than type:



Measures of Central Tendency

Q. 1. Define the following terms:

- **i. Arithmetic mean**
- **ii. Median**
- **iii. Mode**

(i) Arithmetic mean
Ans. Arithmetic mean is used to represent the entire data by one value. Its value is obtained by adding together all the items and by dividing this total by the number of items. Symbolically:

$\bar{X} = (X_1 + X_2 + X_3 + \ldots\ldots\ldots\ldots\ldots\ldots X_n)/N$

or $\bar{X} = \sum X/N$

Where:

$\bar{X}$ = Arithmetic mean

$\sum X$ = Sum of all the values of variable X, i.e., $X_1 + X_2 + X_3 + \ldots\ldots\ldots\ldots\ldots\ldots X_n$

N = Number of observations

(ii) Median
Ans. The median by definition refers to the middle value in a distribution. According to Connor, " the median of a series is that value of the variable which divides the group into two equal parts, one part comprising values greater, and the other all values less than the median".

Calculation of Median – Individual Observations

- Arrange the data in ascending or descending order.
- If n is odd, median = value of [(N + 1)/2]th item.
- If it is even, then median = mean value of N/2 and [(N/2) + 1]th item.

(iii) Mode

Ans. Mode is that value in a series of observations which occurs with the greatest frequency. For example, the mode of the series 2, 4, 7, 4, 3, 4, 8, 1 would be 4, since this value occurs more frequently than any of the others.

Mean

Calculation of Arithmetic Mean – Individual Observations

Q. 2. The following table gives the monthly income of 5 families in a town:

House No	1	2	3	4	5
Income (Rs.)	780	760	690	750	840

Calculate the arithmetic mean of incomes.

Solution. Let the income be denoted by the symbol X

House No.	**Monthly Income X (Rs.)**
1	780
2	760
3	690
4	750
5	840
N = 5	**$\sum X = 3820$**

Mean = $\sum X/N$
$\sum X = 3820$, N=5
Hence, mean = 3820/5 = 764
Thus, the average income is Rs. 764

Calculation of Arithmetic Mean – Discrete Series

Mean = $(\sum fX)/N$
Where:

f = frequency
X = the variable in question
N = total no. of observations (i.e., ∑ f)

- Multiply the frequency of each row with the variable and obtain the total ∑ fX.
- Divide the total obtained in step 1 by the no. of observations (i.e., the total frequency).

Q. 3. From the following the data of the marks obtained by the 60 students of a class, obtain the average marks.

Marks	No of Students	Marks	No. of Students
20	8	50	10
30	12	60	6
40	20	70	4

Solution. Let the marks be denoted by X and the no. of the students by f.

Marks	No of students	fX
X	**f**	
20	8	160
30	12	360
40	20	800
50	10	500
60	6	360
70	4	280
Total	**N = 60**	**∑ fX = 2,460**

Mean = ∑ fX/N = 2460/60 = 41.
Hence, the average marks = 41.

Calculation of Arithmetic Mean - Continuous Series

Mean = ∑ fm/N
Where:
m = midpoint of various classes.
f = frequency of each class.
N = the total frequency

- Obtain the mid point of each class and denote it by ‘m’.
  Mid point = (Lower Limit + Upper Limit)/2
- Multiply these mid points by the respective frequency of each class and obtain the total ∑ fm.
- Divide the total obtained in Step 2 by the sum of frequencies, i.e. N.

Q.4. From the following data compute arithmetic mean:

Marks	**No of Students**	**Marks**	**No of Students**
0-10	5	30-40	30
10-20	10	40-50	20
20-30	25	50-60	10

Solution.

Marks	**Mid points m**	**No of students f**	**f X m**
0-10	5	5	25
10-20	15	10	150
20-30	25	25	625
30-40	35	30	1050
40-50	45	20	900
50-60	55	10	550
Total		**N = 100**	**∑ fm = 3,300**

Mean = ∑ fm/N = 3300/100 = 33
Hence, the average marks = 33.

Calculation of Arithmetic Mean - Continuous Series

Mean = ∑ fm/N
Where:
m = midpoint of various classes.
f = frequency of each class.
N = the total frequency

- Obtain the mid point of each class and denote it by ‘m’.
  Mid point = (Lower Limit + Upper Limit)/2
- Multiply these mid points by the respective frequency of each class and obtain the total $\sum$ fm.
- Divide the total obtained in Step 2 by the sum of frequencies, i.e. N.

Q.4. From the following data compute arithmetic mean:

Marks	No of Students	Marks	No of Students
0-10	5	30-40	30
10-20	10	40-50	20
20-30	25	50-60	10

Solution.

Marks	Mid points m	No of students f	f X m
0-10	5	5	25
10-20	15	10	150
20-30	25	25	625
30-40	35	30	1050
40-50	45	20	900
50-60	55	10	550
Total		**N = 100**	**$\sum$ fm = 3,300**

Mean = $\sum$ fm/N = 3300/100 = 33
Hence, the average marks = 33.

Q. 5. The average marks obtained by students of section A of a class is 80 and that of their counterparts from section B was 70. The mean marks obtained by students from both sections A and B was 78. Find the percentage of students from section A. ***(Dec. 2000)***

Solution. Let the mean marks obtained by students of section A = x_1
Let the mean marks obtained by students of section B = x_2
Let the mean marks obtained by students from both sections A and B = x
Here, $x_1 = 80$, $x_2 = 70$, $x = 78$
$x = (n_1 X x_1 + n_2 X x_2)/(n_1 + n_2)$....... (i)

Putting the values of x, x_1, and x_2 in equation (i), we get
$78 = (n_1 X 80 + n_2 X 70)/(n_1 + n_2)$
or $8 n_2 = 2 n_1$
or $n_1/n_2 = 4/1$

Therefore, the percentage of students from section A = [4/(4 + 1)] X 100 = 80%.

Median

Calculation of Median – Individual Observations

Arrange the data in ascending or descending order.
If n is odd, median = value of [(N + 1)/2]th item.
If it is even, then median = mean value of N/2 and [(N/2) + 1]th item.

Q. 6. Obtain the value of median from the following data:

15 6 16 8 22 21 9 18 25

Solution. Arranging the data in ascending order.

6 8 9 15 16 18 21 22 25

Median = Size of [(9+1)/2]th item.
Size of 5^{th} item = 16.

Calculation of Median – Discrete series

Q. 7. From the following data, find the value of median:

x	8	5	6	10	9	4	7
f	6	4	5	8	9	6	4

Solution. To locate the middle items, we first compute the cumulative frequencies after arranging the values of x in ascending order.

x	**f**	**Cumulative frequency (cf)**
4	6	6
5	4	10
6	5	15

7	4	19
8	6	25
9	9	34
10	8	42
Total	**42**	

Number of observations is 42 (even).
There are two middle items, (42/2)th and [(42/2) + 1]th, i.e., 21^{st} and 22^{nd} items. The value of each of them is 8, since the value of each item 20^{th} to 25^{th} is 8.
Therefore, median = mean of 21^{st} and 22^{nd} items.
(8 + 8)/2 = 8.

Calculation of Median: Continuous series

Median = L + [(h/f)(N/2 - C)]
Where :
L = lower Limit of the median class
h = magnitude of the median class
f = frequency of the median class
C = cumulative frequency of the class preceding the median class.

Q. 8. Compute median from the following data:

Mid value	Frequency	Mid value	Frequency
115	6	165	60
125	25	175	38
135	48	185	22
145	72	195	3
155	116		

Solution.

Class intervals	f	Cumulative frequency (cf)
110-120	6	6
120-130	25	31
130-140	48	79
140-150	72	151
150-160	116	267
160-170	60	327
170-180	38	365
180-190	22	387
190-200	3	390
Total	**390**	

N = 390
N/2 = 390/2 = 195th item.
Therefore, the median class is 150-160

L = 150, N/2 = 195, h = 10, f =116, C = 151
Median = 150 + [(10/116) X (195 - 151)] = 150 + 3.79 = 153.79.

Mode

Calculation of mode - Individual series

Q. 9. Find the mode of the following array of an individual series of scores.

7 7 10 12 12 12 11 13 13 17

Solution. We see that the observation 12 has the maximum frequency 3. Therefore, 12 is the mode.

Calculation of mode - Discrete series

Q. 10. Calculate mode from the data given below:

Marks	**Frequency**
15	2
18	1
19	1
20	1
22	1
25	3
28	1

Here, marks 25 occurs maximum number of times (i.e., 3).
Hence, the modal marks are 25 or mode = 25.

Calculation of mode – Continuous series

Steps

1. Locate the maximum frequency, the corresponding class is the modal class.

2. Determine the value of the mode by applying the following formula:

   Mode = $L + [h(f_0 - f_{-1})/(2f_0 - f_{-1} - f_{+1})]$

Where:
L = lower limit of the modal class.
h = class interval of the modal class.
f_0 = frequency of the modal class.
f_{-1} = frequency of the class preceding the modal class.
f_{+1} = frequency of the class succeeding the modal class.

Q. 11. The income of 80 families are given below: (Dec. 2001)

Income (in Rs.)	**No. of families**
4000 - 6000	8
6000 - 8000	24
8000 - 10000	32
10000 - 12000	16

Find the mode.

Solution. By inspection, the maximum frequency is 32.
So 8000 - 10000 is the modal class.
$L = 8000$, $h = 2000$, $f_0 = 32$, $f_{-1} = 24$, $f_{+1} = 16$

Mode = $L + [h(f_0 - f_{-1})/(2f_0 - f_{-1} - f_{+1})]$
$= 8000 + \{2000(32 - 24)/[(2 \text{ X } 32) - 24 - 16]\}$
$= 8666.67$

Hence, the mode income is Rs. 8666.67.

Measures of Dispersion

Q. 1. Define the following terms:

i. **Range**
ii. **Standard Deviation**
iii. **Variance**
iv. **Mean Deviation**

(i) Range
Ans. Range is the difference between the value of the smallest item and the value of the largest item included in the distribution. Range = L - S
Where:
L = Largest item
S = Smallest item
Coefficient of Range = (L - S)/(L + S)

(ii) Standard Deviation
Ans. Standard deviation measures the absolute dispersion or variability of a distribution. It is denoted by σ.

(iii) Variance
Ans. Variance is the average squared deviations from the arithmetic mean. It is denoted by σ^2.

(iv) Mean Deviation
Ans. It is the average difference between the items in a distribution from the median or mean of the series. The mean deviation is also known as the average deviation.

Range

Q. 2. The following are the prices of shares of TCS Ltd. from Monday to Wednesday:

Days	**Price (Rs.)**
Monday	200
Tuesday	210
Wednesday	208

Calculate range and its coefficient.

Solution. Range = L - S
Here, L = 210 and S = 200
Therefore, Range = 210 - 200 = Rs.10
Coefficient of Range = (210 - 200)/(210 + 200) = 10/410 = 0.02.

Standard Deviation

Q. 3. The following table gives weights (in kilograms) of 100 students, randomly selected from a college: (June 2002)

Weight in Kilograms	Number of students
45-50	01
50-55	05
55-60	21
60-65	43
65-70	22
70-75	06
75-80	02

Calculate the standard deviation of the above frequency distribution.

Solution.

Weight in Kilograms	Mid point (M)	No. of Students (f)	f X M	x = M - mean	x^2	f X x^2
45-50	47.5	01	47.5	-15.3	234.09	234.09
50-55	52.5	05	262.5	-10.3	106.09	530.45
55-60	57.5	21	1207.5	-5.3	28.09	589.89
60-65	62.5	43	2687.5	-0.3	0.27	11.61
65-70	67.5	22	1485	4.7	22.09	485.98
70-75	72.5	06	435	9.7	94.09	564.54
75-80	77.5	02	155	14.7	216.09	432.18
Total		**$\sum$ N = 100**	**$\sum$ fM = 6280**			**$\sum fx^2 = 2848.74$**

Mean = $\sum$ fM/$\sum$ N = 6280/100 = 62.8

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum N}}$$

$$\text{or } \sigma = \sqrt{\frac{2848.74}{100}}$$

or σ = 5.337

Q. 4. Calculate the standard deviation of the following grouped frequency distribution. (June 2000)

Height in inches **Number of students**

60-65	12
65-70	35
70-75	11
75-80	2
80-85	0

Solution.

Height in inches	Mid point (M)	No. of Students (f)	f X M	x = M - mean	x^2	f X x^2
60-65	62.5	12	750	-5.25	27.5625	330.75
65-70	67.5	35	2362.5	-0.25	0.0625	2.1875
70-75	72.5	11	797.5	4.75	22.5625	248.1875
75-80	77.5	2	155	9.75	95.0625	190.125
80-85	82.5	0	0	14.75	217.5625	0
Total		**$\sum N = 60$**	**$\sum fM = 4065$**			**$\sum fx^2 = 771.25$**

Mean = $\sum fM / \sum N = 4065/60 = 67.75$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum N}}$$

$$\text{or } \sigma = \sqrt{\frac{771.25}{60}}$$

or $\sigma = 3.58$

Q. 5. For a frequency distribution of marks in History of 200 candidates, the mean and standard deviation (s.d.) were found to be 40 and 15 respectively. Later it was discovered that the score 43 was misread as 53 in distribution. Find the correct mean and standard deviation corresponding to the correct distribution. (June 2002, Dec. 99)

Solution. Given, N = 200, $\bar{x} = 40$, and $\sigma = 15$

Calculation of Correct Mean

$\bar{X} = \sum X/N$
or $\sum X = N\bar{X}$
or $\sum X = 200 \times 40 = 8000$
But this is not correct $\sum X$ because the score 43 was misread as 53 in the distribution.
Correct $\sum X = 8000 - 53 + 43 = 7990$
Correct $\bar{X} = 7990/200 = 39.95$

Calculation of Correct Standard Deviation

$\sigma^2 = (\sum X^2/N) - (\bar{X})^2$
or $(15)^2 = (\sum X^2/200) - (40)^2$
or $\sum X^2 = 365000$
Correct $\sum X^2 = 365000 - (53)^2 + (43)^2 = 364040$

Correct $\sigma = \sqrt{[(\text{Correct} \sum X^2/N) - (\text{Correct } \bar{X})^2]}$

or Correct $\sigma = \sqrt{[(364040/200) - (39.95)^2]}$
$= 14.97$

Thus, the correct mean is 39.95 and correct standard deviation is 14.97.

Variance

Q. 6. Calculate the variance for the class-frequency distribution given below: (Dec. 2001)

Marks obtained	Number of students
0-10	15
10-20	20
20-30	25
30-40	17
40-50	12

Solution.

Marks obtained	Mid point (M)	Number of students (f)	f X M	x = M - mean	x^2	f X x^2

0-10	5	15	75	-18.98	360.2404	5403.606
10-20	15	20	300	-8.98	80.6404	1612.808
20-30	25	25	625	1.02	1.0404	26.01
30-40	35	17	595	11.02	121.4404	2064.4868
40-50	45	12	540	21.02	441.8404	5302.0848
Total		**$\sum N = 89$**	**$\sum fM = 2135$**			**$\sum fx^2 = 14408.9956$**

Mean = $\sum fM / \sum N = 2135/89 = 23.98$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum N}}$$

$$\text{or } \sigma = \sqrt{\frac{14408.9956}{89}}$$

or $\sigma = 12.72$

Variance, $\sigma^2 = 160.79$

Q. 7. Following table gives the height (in inches) of the employees of an organization: (June 2001)

Height in inches	Number of employees
50-60	05
60-70	35
70-80	08
80-90	02

Calculate the variance of the above class-distribution.

Solution.

Height in inches	Mid point (M)	Number of employees (f)	f X M	x = M - mean	x^2	f X x^2
50-60	55	05	275	-11.4	129.96	649.8
60-70	65	35	2275	-1.4	1.96	68.6
70-80	75	08	600	8.6	73.96	591.68

80-90	85	02	170	18.6	345.96	691.92
Total		**$\sum N = 50$**	**$\sum fM = 3320$**			**$\sum fx^2 = 2002$**

Mean = $\sum fM / \sum N = 3320/50 = 66.4$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum N}}$$

or

$$\sigma = \sqrt{\frac{2002}{50}}$$

or $\sigma = 6.32$

Variance, $\sigma^2 = 39.94$

Q. 8. The following table gives the height (in inches) of 100 students, randomly selected from a college: (Dec. 2002)

Height in inches	Number of students
57-60	02
60-63	06
63-66	20
66-69	45
69-72	21
72-75	05
75-78	01

Calculate the variance of the above frequency distribution.

Solution.

Height in inches	Mid point (M)	Number of students (f)	f X M	x = M - mean	x^2	f X x^2
57-60	58.5	02	117	-8.88	78.8544	157.7088
60-63	61.5	06	369	-5.88	34.5744	207.4464
63-66	64.5	20	1290	-2.88	8.2944	165.888

66-69	67.5	45	3037.5	0.12	0.0144	0.648
69-72	70.5	21	1480.5	3.12	9.7344	204.4244
72-75	73.5	05	367.5	6.12	37.4544	187.272
75-78	76.5	01	76.5	9.12	83.1744	83.1744
Total		**$\sum$ N = 100**	**$\sum$ fM = 6738**			**$\sum fx^2 = 1006.56$**

Mean = $\sum$ fM/$\sum$ N = 6738/100 = 67.38

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum N}}$$

or

$$\sigma = \sqrt{\frac{1006.56}{100}}$$

or $\sigma = 3.17$
Variance, $\sigma^2 = 10.0489$

Q. 9. The mean of 5 observations is 4.4 and the variance is 8.24. If three of the five observations are 1, 2 and 6, find the other two. (Dec. 2000)

Solution. Given, N = 5, $\bar{X} = 4.4$, and $\sigma^2 = 8.24$
$\bar{X} = \sum X/N$
or $\sum X = N\bar{X}$
or $\sum X = 5 \times 4.4 = 22$

Let the two missing items be p and q
Therefore, 1 + 2 + 6 + p + q = 22
or p + q = 13 .... (i)

$\sigma^2 = (\sum X^2/N) - (\bar{X})^2$
or $8.24 = (\sum X^2/5) - (4.4)^2$
or $\sum X^2 = 138$

Also, $\sum X^2 = 1^2 + 2^2 + 6^2 + p^2 + q^2$
or $p^2 + q^2 = 138 - 41$
or $p^2 + q^2 = 97$

$(p + q)^2 = p^2 + q^2 + 2pq$
or $(13)^2 = 97 + 2pq$
or $pq = 36$

$(p - q)^2 = p^2 + q^2 - 2pq$
or $(p - q)^2 = 97 - (2 X 36)$
or $p - q = 5$ .... (ii)

Adding equation (i) and (ii), we get
$p = 9$ and $q = 4$
Thus, the two missing values are 9 and 4.

Mean Deviation

Mean Deviation

Calculation of Mean Deviation – Individual Observations

Mean Deviation or M.D. = $\sum |D|/N$
Where |D| within parallel line read as MOD(X - mean) is the absolute value of the deviation from mean after ignoring signs.

(i) Compute the mean of series.
(ii) Calculate the deviation of item from mean ignoring sign and denote these deviations by |D|.
(iii) Calculate the total of these deviations, i.e. |D|.
(iv) Divide the total obtained in step (iii) by the number of observations.

Calculation of Mean Deviation – Discrete Series

Mean Deviation or M.D. = $\sum f|D|/N$
Where, |D| within parallel line read as MOD(X - mean) is the absolute value of the deviation from mean after ignoring signs.

(i) Compute the mean of series.
(ii) Calculate deviation of the item from mean ignoring sign and denote these deviations by |D|.
(iii) Multiply these deviations by the respective frequencies and obtain the total $\sum f|D|$.
(iv) Divide the total obtained in step (iii) by the number of observations. This gives the value of mean deviation.

Calculation of Mean Deviation – Continuous Series

For calculating the mean deviation in continuous series, the procedure remains the same as discussed above. The only difference is that here we have to obtain the mid-point of the various classes and take deviations of these points from median. The formula is the same, i.e.,
M.D. = $\sum f|D|/N$

Q. 10. Compute the mean deviation for the following set of data.

Marks	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	5	14	10	8	6	4

Solution.

Marks	**Midpoint (M)**	**Frequency (f)**	**f X M**	**|D| = M – mean**	**f X |D|**
20-30	25	5	125	21.7	108.5
30-40	35	14	490	11.7	163.8
40-50	45	10	450	1.7	17.0
50-60	55	8	440	8.3	66.4
60-70	65	6	390	18.3	109.8
70-80	75	4	300	28.3	113.2
		∑ N = 47	**∑ fM = 2195**		**∑ f|D| = 578.7**

Mean = ∑ fM/N = 2195/47 = 46.70
M.D. = ∑ f|D|/N = 578.7/47 = 12.31

Coefficient of Variation

Coefficient of variation (C.V.) is the relative measure of dispersion.
C.V. = (Standard Deviation/Mean) X 100

Q. 11. The wage pattern of the workers of a firm is given below:

	Firm
No. of workers	400
Average monthly income	350
Standard Deviation	8

Find the coefficient of variation.

Solution.

C.V. = (8/350) X 100 = 2.28%

Coefficient of Dispersion

1. Based on Standard Deviation
C.D. = Standard Deviation / Mean

2. Based on Mean Deviation
C.D. = Mean Deviation/Average

Correlation and Regression

Correlation

DEFINITION:

1. "Correlation analysis deals with the association between two or more variables." - **Simpson & Kafka.**

2. "Correlation analysis attempts to determine the 'degree of relationship' between variables." **- Ya Lun Chou.**
3. "Correlation is an analysis of the covariation between two or more variables." **- A.M. Tuttle.**

Karl Pearson's coefficient of correlation

$$r = \frac{[n \sum xy - (\sum x \times \sum y)]}{\sqrt{\{[n \sum x^2 - (\sum x)^2] \times [n \sum y^2 - (\sum y)^2]\}}}$$

Where:
x = variable 1
y = variable 2
n = number of pair of scores
r = coefficient of linear correlation

Q. 1. Calculate the correlation coefficient for the following data: (June 2002)

x	8	12	15	20	24	27	32
y	30	24	36	44	56	64	72

Solution.

x	**y**	x^2	y^2	**x X y**
8	30	64	900	240
12	24	144	576	288
15	36	225	1296	540
20	44	400	1936	880
24	56	576	3136	1344
27	64	729	4096	1728
32	72	1024	5184	2304
$\sum x = 138$	**$\sum y = 326$**	**$\sum x^2 = 3162$**	**$\sum y^2 = 17124$**	**$\sum xy = 7324$**

Here, n = 7

$$r = \frac{[n \sum xy - (\sum x \times \sum y)]}{}$$

$$\sqrt{\{[n \sum x^2 - (\sum x)^2] \text{ X } [n \sum y^2 - (\sum y)^2]\}}$$

$$\text{or } r = \frac{[(7 \text{ X } 7324) - (138 \text{ X } 326)]}{\sqrt{\{[(7 \text{ X } 3162) - (138)^2] \text{ X } [(7 \text{ X } 17124) - (326)^2]\}}}$$

or r = 0.969

Note: Coefficient of correlation lies between +1 and -1.

Q. 2. Calculate the correlation coefficient for the following data: (June 2001)

x	y
15	9
10	12
5	18
12	10
17	5
18	2

Solution.

x	y	x^2	y^2	x X y
15	9	225	81	135
10	12	100	144	120
5	18	25	324	90
12	10	144	100	120
17	5	289	25	85
18	2	324	4	36
$\sum x = 77$	$\sum y = 56$	$\sum x^2 = 1107$	$\sum y^2 = 678$	$\sum xy = 586$

Here, n = 6

$$r = \frac{[n \sum xy - (\sum x \text{ X } \sum y)]}{}$$

$$\sqrt{\{[n \sum x^2 - (\sum x)^2] \text{ X } [n \sum y^2 - (\sum y)^2]\}}$$

$$\text{or } r = \frac{[(6 \text{ X } 586) - (77 \text{ X } 56)]}{\sqrt{\{[(6 \text{ X } 1107) - (77)^2] \text{ X } [(6 \text{ X } 678) - (56)^2]\}}}$$

or r = - 0.976

Q. 3. For a local departmental store, a table of Annual Expenditure for Advertisement and corresponding sales (for that year) is given for a number of years as below. Find the coefficient of correlation for advertisement expenditure (as independent variable) and sales. (June 2000)

Advertisement Expenditure in Thousand Rupees	0.45	0.55	0.72	0.83	1.25
Sales in Thousand Rupees	70	81	83	85	91

Solution.
Let advertisement expenditure (thousand rupees) = x
Let sales (thousand rupees) = y

x	**y**	x^2	y^2	**x X y**
0.45	70	0.2025	4900	31.5
0.55	81	0.3025	6561	44.55
0.72	83	0.5184	6889	59.76
0.83	85	0.6889	7225	70.55
1.25	91	1.5625	8281	113.75
$\sum x = 3.80$	$\sum y = 410$	$\sum x^2 = 3.2748$	$\sum y^2 = 33856$	$\sum xy = 320.11$

Here, n = 5

$$r = \frac{[n \sum xy - (\sum x \text{ X } \sum y)]}{\sqrt{\{[n \sum x^2 - (\sum x)^2] \text{ X } [n \sum y^2 - (\sum y)^2]\}}}$$

$$\text{or } r = \frac{[(5 \times 320.11) - (3.80 \times 410)]}{\sqrt{\{[(5 \times 3.278) - (3.80)^2] \times [(5 \times 33856) - (410)^2]\}}}$$

or r = 0.89

Q. 4. A computer while calculating the correlation coefficient between 20 pairs of two variables x and y obtain the following results: (Dec. 2001)
$n = 20, \sum x = 100, \sum y = 80, \sum x^2 = 520, \sum y^2 = 360, \sum xy = 420$

It was later discovered at the time of checking that he had copied down two pairs as:

x	**y**		**x**	**y**
6	4	**While the correct values were:**	8	12
8	6		6	8

Obtain the correct value of correlation coefficient.

Solution.
Here, $n = 20, \sum x = 100, \sum y = 80, \sum x^2 = 520, \sum y^2 = 360, \sum xy = 420$

Incorrect pair		**Correct pair**	
x	**y**	**x**	**y**
6	4	8	12
8	6	6	8

Correct $\sum x = 100 + (8 - 6) + (6 - 8) = 100$ (no change)
Correct $\sum y = 80 + (12 - 4) + (8 - 6) = 90$
Correct $\sum x^2 = 520 + (8^2 - 6^2) + (6^2 - 8^2) = 520$ (no change)
Correct $\sum y^2 = 360 + (12^2 - 4^2) + (8^2 - 6^2) = 516$
Correct $\sum xy = 420 + [(8 \times 12) - (6 \times 4)] + [(6 \times 8) - (8 \times 6)] = 492$

$$r = \frac{[(20 \times 492) - (100 \times 90)]}{}$$

$$\sqrt{\{[(20 \text{ X } 520) - (100)^2] \text{ X } [(20 \text{ X } 516) - (90)^2]\}}$$

or r = 0.893

Regression

DEFINITION:

1. "Regression is the measure of the average relationship between two or more variables in terms of the original units of the data." **- Blair**
2. "One of the most frequently used technique in economics and business research, to find a relation between two or more variables that are related casually, is regression analysis." **- Taro Yamane**

The regression equation of y on x is expressed as follows:
y = a + bx
where:
y = dependent variable
x = independent variable
a and b are constants
$a = (1/n) (\sum y - b\sum x)$

$$b = \frac{[n \sum xy - (\sum x \text{ X } \sum y)]}{[n \sum x^2 - (\sum x)^2]}$$

Q. 5. Fit a straight line to the data given by the following table: (June 2002)

Independent Variable	**Dependent Variable**
x	**y**
2	3
4	17
6	38
7	49
9	80
11	120

Solution.

x	y	x^2	x X y
2	3	4	6
4	17	16	68
6	38	36	228
7	49	49	343
9	80	81	720
11	120	121	1320
$\sum x = 39$	**$\sum y = 307$**	**$\sum x^2 = 307$**	**$\sum xy = 2685$**

Equation of straight line:

y = a + bx

Here, n = 6

$$b = \frac{[n \sum xy - (\sum x \text{ X } \sum y)]}{[n \sum x^2 - (\sum x)^2]}$$

$$\text{or } b = \frac{[(6 \text{ X } 2685) - (39 \text{ X } 307)]}{[(6 \text{ X } 307) - (39)^2]}$$

or b = 12.88

a = (1/6) X [(307 - (12.88 X 39)] = -32.55

Therefore, y = -32.55 + 12.88x

Q. 6. Fit a straight line to the data given by the following table: (June 2001)

Independent Variable	Dependent Variable
x	**y**
1	6
2	5
4	9
5	11
6	13
8	17

Solution.

x	y	x^2	x X y

1	6	1	6
2	5	4	10
4	9	16	36
5	11	25	55
6	13	36	78
8	17	64	136
$\sum x = 26$	**$\sum y = 61$**	**$\sum x^2 = 146$**	**$\sum xy = 321$**

Equation of straight line:

y = a + bx
Here, n = 6

$$b = \frac{[n \sum xy - (\sum x \text{ X } \sum y)]}{[n \sum x^2 - (\sum x)^2]}$$

$$\text{or } b = \frac{[(6 \text{ X } 321) - (26 \text{ X } 61)]}{[(6 \text{ X } 146) - (26)^2]}$$

or b = 1.7
a = (1/6) X [(61 - (1.7 X 26)] = 2.8
Therefore, y = 2.8 + 1.7x

Q. 7. In a partially destroyed laboratory record of an analysis of correlation data, only the following results are legible: (Dec. 2000)
Variance of x = 9
Regression equations:
8x - 10y + 66 = 0 of y on x
40x - 18y - 214 = 0 of x on y

Find out
(i) the mean values of x and y
(ii) the correlation coefficient between x and y

Solution.

(i) Mean values of x and y

8x - 10y = -66 ....(1)
40x - 18y = 214 ....(2)

Multiplying equation (1) by 5 and subtracting (2) from (1), we get

$$\begin{array}{lrcl} & 40x & -50y & = -330 \\ & 40x & -18y & = 214 \\ - & & + & - \\ \hline & & -32y & = -544 \end{array}$$

or y = 17

Substituting the value of y in equation (1)
8x - 10 X 17 = -66
or x = 13

Therefore, mean values of x and y are 13 and 17 respectively.

(ii) Correlation coefficient between x and y

From equation (1)
x = -66/8 + (10/8)y
Therefore, regression coefficient of x on y (b_{xy}) = 10/8 = 1.25

From equation (2)
y = -214/18 + (40/18)x
Therefore, regression coefficient of y on x (b_{yx}) = 40/18 = 2.22

Since both the regression coefficients are exceeding 1, our assumption is wrong. Hence, the equation (1) is the equation of y on x.

From equation (1)
-10y = -8x - 66
or y = (8/10)x + 6.6
Therefore, b_{yx} = 8/10 = 0.8

From equation (2)
b_{xy} = 18/40 = 0.45

$r^2 = (b_{xy} \text{ X } b_{yx})$
r^2 = (0.45 X 0.8)
or r = ± 0.6
Since both the regression coefficients are positive, we take r = + 0.6
Therefore, correlation coefficient between x and y is 0.6.

Q. 8. Fit a straight line to the data given by the following table: (Dec. 2002)

Independent Variable	**Dependent Variable**
y	**z**
1	1
3	8
4	17
6	34
7	52
9	78

Solution.

y	**z**	**y^2**	**y X z**
1	1	1	1
3	8	9	24
4	17	16	68
6	34	36	204
7	52	49	364
9	78	81	702
$\sum y = 30$	**$\sum z = 190$**	**$\sum y^2 = 192$**	**$\sum yz = 1363$**

Equation of straight line:

y = a + bx
Here, n = 6

$$b = \frac{[n \sum yz - (\sum y \text{ X } \sum z)]}{[n \sum y^2 - (\sum y)^2]}$$

$$\text{or } b = \frac{[(6 \text{ X } 1363) - (30 \text{ X } 190)]}{[(6 \text{ X } 192) - (30)^2]}$$

or b = 9.83
a = [(1/n) X ($\sum$ z - b$\sum$ y)]
or a = (1/6) X [(190 - (9.83 X 30)] = -17.48
Therefore, y = -17.48 + 9.83x

Probability

Q. 1. Define Probability.

Ans. The probability of a given event is an expression of likelihood of occurence of an event. A probability is a number which ranges from 0 (zero) to 1 (one) - zero for an event which cannot occur and 1 for an event which is certain to occur.

Q. 2. Two coins are tossed. Find the probability of getting:

1. **One head and one tail**
2. **Exactly two heads**

Solution. When two coins are tossed, the total number of possible pairs is:
(H, H), (H, T), (T, T), (T, H)
So, n = 4

1) Favourable number of cases = 2
Therefore, required probability = 2/4 = 1/2

2) Favourable number of cases = 1
Therefore, required probability = 1/4

Q. 3. A bag contains 5 white and 3 black balls. Two balls are drawn at random without replacement. Determine the probability of getting both the balls black.

Solution. We have to get 2 black balls from 3 black balls.

Favourable cases = $3C_2$
Total number of cases = $8C_2$

Required probability = $\dfrac{3C_2}{8C_2}$

or Probability = $\dfrac{(3!)/[(2!) \text{ X } (3 - 2)!]}{(8!)/[(2!) \text{ X } (8 - 2)!]}$

or Probability = 3/28

Q. 4. A jar contains 7 red balls, 5 green balls, 4 blue balls, and 3 white balls. A sample of size 7 balls is selected at random without replacement. Find the probability that the sample contains 2 red balls, 2 green balls, 2 blue balls, and 1 white ball. (June 2002)

Solution. Total number of balls = 7 + 5 + 4 + 3 = 19
Favourable cases = $7C_2$ X $5C_2$ X $4C_2$ X $3C_1$
A sample of size 7 balls is selected at random without replacement.
Therefore, total number of cases = $19C_7$

Required probability = $$\frac{7C_2 \text{ X } 5C_2 \text{ X } 4C_2 \text{ X } 3C_1}{19C_7}$$

or Probability = $$\frac{\{(7!)/[(2!)(5!)]\} \text{ X } \{(5!)/[(2!)(3!)]\} \text{ X } \{(4!)/[(2!)(2!)]\} \text{ X } \{(3!)/[(1!)(2!)]\}}{\{(19!)/[(7!)(12!)]\}}$$

or Probability = 0.075

Q. 5. A jar contains 6 red balls, 4 green balls, 3 blue balls, and 2 white balls. A sample of size 6 balls is selected at random without replacement. Find the probability that the sample contains 2 red balls, 2 green balls, 1 blue ball, and 1 white ball. (June 2001)

Solution. Total number of balls = 6 + 4 + 3 + 2 = 15
Favourable cases = $6C_2$ X $4C_2$ X $3C_1$ X $2C_1$
A sample of size 6 balls is selected at random without replacement.
Therefore, total number of cases = $15C_6$

Required probability = $$\frac{6C_2 \text{ X } 4C_2 \text{ X } 3C_1 \text{ X } 2C_1}{15C_6}$$

or Probability = $$\frac{\{(6!)/[(2!)(4!)]\} \text{ X } \{(4!)/[(2!)(2!)]\} \text{ X } \{(3!)/[(1!)(2!)]\} \text{ X } \{(2!)/[(1!)(1!)]\}}{\{(15!)/[(6!)(9!)]\}}$$

or Probability = 0.107

Q. 6. What is the probability that a leap year selected at random contains 53 Sundays? (Dec. 98)

Solution. A leap year consists of 366 days and, therefore, contains 52 complete weeks and 2 days extra. These 2 days may make the following 7 combinations:
1. Monday and Tuesday
2. Tuesday and Wednesday
3. Wednesday and Thursday
4. Thursday and Friday
5. Friday and Saturday
6. Saturday and Sunday
7. Sunday and Monday

Of these seven equally likely cases, only the last two are favourable. Hence, the required probability is 2/7.

Q. 7. The probability that a student passes statistics is 2/3, and the probability that he passes mathematics is 4/9. If the probability of passing both the courses is 1/4, what is the probability that the student passes at least one of these courses? (Jan. 2001, Dec. 2000)

Solution.
Let A = (the event that the student passes statistics)
B = (the event that the student passes mathematics)
Given, P(A) = 2/3, P(B) = 4/9, $P(A \cap B) = 1/4$
$(A \cup B)$ = The event that the student passes at least one of the courses
$P(A \cup B) = 2/3 + 4/9 - 1/4 = 31/39$
Hence, the required probability is 31/39.

Q. 8. If the probabilities that a person purchasing a new car will choose green, white, red or blue colour are 0.08, 0.09, 0.15 and 0.21 respectively, then what is the probability that a given buyer will choose a new car which has any one of these colours? (June 2001)

Solution. Here the event of buying a car of particular colour is mutually exclusive, i.e., the buyer will purchase either a car of any of the four colours.
Let A = an event that the buyer selects a green car
B = an event that the buyer selects a white car
C = an event that the buyer selects a red car
D = an event that the buyer selects a blue car
Therefore, $P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D)$
or $P(A \cup B \cup C \cup D) = 0.08 + 0.09 + 0.15 + 0.21 = 0.53$
Hence, the required probability is 0.53.

Q. 9. Two dice are thrown. Find the probability that sum of the numbers on two dice is 9, given that the first die shows 6. (Dec. 2001)

Solution. Two dice can be thrown in 6 X 6 = 36 ways.
A total of 9 can be obtained as
(6, 3), (3, 6), (5, 4), (4, 5), i.e., in 4 ways.

Required probability = 4/36 or 1/9.

Q. 10. The probability of a college student being male is 1/3 and that of being female is 2/3. The probability that a male student completes the course is 5/14 and that of a female student completes is 5/11. A student is selected at random and is found to have completed the course. What is the probability that the student is a male? (June 2000)

Solution. This problem is based on Bayes theorem.
Let A = an event that the student completes the course
B_1 = an event that the student selected is male
B_2 = an event that the student selected is female

Therefore, $P(B_1) = 1/3$, $P(B_2) = 2/3$
$P(A \mid B_1) = 5/14$, $P(A \mid B_2) = 5/11$

$$P(B_1 \mid A) = \frac{P(B_1)\, P(A \mid B_1)}{P(B_1)\, P(A \mid B_1) + P(B_2)\, P(A \mid B_2)}$$

$$\text{or } P(B_1 \mid A) = \frac{(1/3) \text{ X } (5/14)}{[(1/3) \text{ X } (5/14)] + [(2/3) \text{ X } (5/11)]}$$

or $P(B_1 \mid A) = 0.28$
Thus, the probability that the student being male is 0.28.

Q. 11. The probability of a college student being male is 1/3 and that of being female is 2/3. The probability that a male student completes the course is 3/4 and that a female student does it is 1/2. A student is selected at random and is found to have completed the course. What is the probability that the student is a male? (Dec. 2001)

Solution. This problem is based on Bayes theorem.
Let A = an event that the student completes the course
B_1 = an event that the student selected is male
B_2 = an event that the student selected is female

Therefore, $P(B_1) = 1/3$, $P(B_2) = 2/3$
$P(A \mid B_1) = 3/4$, $P(A \mid B_2) = 1/2$

$$P(B_1 \mid A) = \frac{P(B_1)\, P(A \mid B_1)}{P(B_1)\, P(A \mid B_1) + P(B_2)\, P(A \mid B_2)}$$

$$\text{or } P(B_1 \mid A) = \frac{(1/3) \text{ X } (3/4)}{[(1/3) \text{ X } (3/4)] + [(2/3) \text{ X } (1/2)]}$$

or $P(B_1 \mid A) = 0.428$
Thus, the probability that the student being male is 0.428.

Q. 12. In a bolt factory machines A, B and C manufacture respectively 25, 30 and 40 percent of the total. Out of their total outputs 5, 4 and 2 percent are defective. A bolt is drawn from the produce at random and is found to be defective. What is the probability that it is manufactured by (i) factory A (ii) factory C? (June 99)

Solution. This problem is based on Bayes theorem.
Let A = an event that the output is defective
B_1 = an event that bolt is manufactured by machine A
B_2 = an event that bolt is manufactured by machine B
B_3 = an event that bolt is manufactured by machine C

Therefore, $P(B_1) = 25/100$, $P(B_2) = 35/100$, $P(B_3) = 40/100$
$P(A \mid B_1) = 5/100$, $P(A \mid B_2) = 4/100$, $P(A \mid B_3) = 2/100$

$$\text{(i) } P(B_1 \mid A) = \frac{P(B_1)\, P(A \mid B_1)}{P(B_1)\, P(A \mid B_1) + P(B_2)\, P(A \mid B_2) + P(B_3)\, P(A \mid B_3)}$$

$$\text{or } P(B_1 \mid A) = \frac{(25/100) \text{ X } (5/100)}{[(25/100) \text{ X } (5/100)] + [(35/100) \text{ X } (4/100)] + [(40/100) \text{ X } (2/100)]}$$

or $P(B_1 \mid A) = 25/69$

(ii) $$P(B_3 \mid A) = \frac{P(B_3)\,P(A \mid B_3)}{P(B_1)\,P(A \mid B_1) + P(B_2)\,P(A \mid B_2) + P(B_3)\,P(A \mid B_3)}$$

or $$P(B_1 \mid A) = \frac{(40/100) \text{ X } (2/100)}{[(25/100) \text{ X } (5/100)] + [(35/100) \text{ X } (4/100)] + [(40/100) \text{ X } (2/100)]}$$

or $P(B_3 \mid A) = 16/69$

Q. 13. In a bolt factory machines A, B and C manufacture respectively 30, 35 and 35 percent of the total. Out of their total outputs 3, 4 and 3 percent are defective. A bolt is drawn at random and is found to be defective. What is the probability that it is manufactured by (i) factory A (ii) factory B? (June 2002)

Solution.
Let X, Y and Z denote the event that a bolt is manufactured by the machine A, B and C respectively.
Let S denotes the event that the bolt is drawn now.

$P(X \cap S) = (30/100) \text{ X } (3/100) = 90/10000$
$P(Y \cap S) = (35/100) \text{ X } (4/100) = 140/10000$
$P(Z \cap S) = (35/100) \text{ X } (3/100) = 105/10000$

$P(S) = [(90/10000) + (140/10000) + (105/10000)]$
or $P(S) = 335/10000$

(i) $$P(X \mid S) = \frac{P(X \cap S)}{P(S)}$$

or $$P(X \mid S) = \frac{(90/10000)}{(335/10000)}$$

or $P(X \mid S) = 18/67$

(ii) $$P(Y \mid S) = \frac{P(Y \cap S)}{}$$

P(S)

$$\text{or } P(Y \mid S) = \frac{(140/10000)}{(335/10000)}$$

or P(Y | S) = 28/67

Q. 14. Find the expected value of the number X shown on the face of a dice, when the dice is thrown. The dice is unbiased. (Note : Face value of a dice is 1, 2, 3, 4, 5 or 6) (Jan. 2001)

Solution. X can take values 1, 2, 3, 4, 5, 6 each with a probability 1/6.

$$E(X) = \sum_{i=1}^{6} x_i P(x = x_i)$$

or E(X) = [1 X (1/6)] + [2 X (1/6)] + [3 X (1/6)] + [4 X (1/6)] + [5 X (1/6)] + [6 X (1/6)]
or E(X) = 3.5

Q. 15. A consignment of eight similar microcomputers to retail outlet contains 3 that are defective. If a firm makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives. (Dec. 98)

Solution. Let X = random variable (i.e., random purchase of 2 defective computers)
So, x can take the values (0, 1, 2)

F[X = x = 0] = P[x = 0] = $({}^3C_0 \text{ X } {}^5C_2)/{}^8C_2$
= 0.36
Therefore, F[x = 0] = 0.36

F[X = x = 1] = P[x = 1] = $({}^3C_1 \text{ X } {}^5C_1)/{}^8C_2$
= 0.53
Therefore, F[x = 1] = 0.53

F[X = x = 2] = P[x = 2] = $({}^3C_2 \text{ X } {}^5C_0)/{}^8C_2$
= 0.11
Therefore, F[x = 2] = 0.11

Q. 16. The probability that a certain kind of component will survive a given shock test is 3/4. Find the probability that exactly two of the next four components tested will survive. (Jan. 2001, Dec. 2000, June 2000)

Solution. Here n = 4, x = 2, p = 3/4
q = 1 - p
or q = [1- (3/4)]
or q = 1/4
Using Binomial Distribution
$P[X = x] = (^nC_x)\ X\ (p^x)\ X\ (q^{n-x})$
$P[X = 2] = (^4C_2)\ X\ (3/4)^2\ X\ (1/4)^2$
= 0.21
Hence, the required probability is 0.21.

Q. 17. Let X be the number of 1's obtained in 15 throws of an unbiased dice. Find its mean and variance. (June 2001)

Solution. X is a binomial variance with p = 1/6, q = [1 - (1/6)] = 5/6, and n = 15
Mean = n X p
or Mean = 15 X (1/6) = 2.5

Variance = n X p X q
or Variance = 15 X (1/6) X (5/6) = 2.09
Therefore, mean = 2.5 and variance = 2.09.

Q. 18. The average number of radioactive particles through a counter during 1 milli second in a laboratory experiment is 3. What is the probability that five particles enter the counter in a given millisecond? (Dec. 2001)

Solution. Given, x = 5 and λ = 3
Using Poisson Distribution
Probability = $[e^{-\lambda}(\lambda)^x]/x!$
= $[e^{-3}(3)^5]/5!$
= 0.1008

Q. 19. In a bulb making factory, three machines A, B and C manufacture respectively 15, 35 and 50 percent of the total. Out of their total outputs 4, 5 and 3 percent are defective. A bulb is drawn from the produce at random and is found to be defective. What is the probability that it is manufactured by (i) factory A (ii) factory C? (Dec. 2002)

Solution. This problem is based on Bayes theorem.
Let A = an event that the output is defective
B_1 = an event that bulb is manufactured by machine A
B_2 = an event that bulb is manufactured by machine B
B_3 = an event that bulb is manufactured by machine C

Therefore, $P(B_1) = 15/100$, $P(B_2) = 35/100$, $P(B_3) = 50/100$
$P(A \mid B_1) = 4/100$, $P(A \mid B_2) = 5/100$, $P(A \mid B_3) = 3/100$

$$\text{(i) } P(B_1 \mid A) = \frac{P(B_1)\, P(A \mid B_1)}{P(B_1)\, P(A \mid B_1) + P(B_2)\, P(A \mid B_2) + P(B_3)\, P(A \mid B_3)}$$

$$\text{or } P(B_1 \mid A) = \frac{(15/100) \text{ X } (4/100)}{[(15/100) \text{ X } (4/100)] + [(35/100) \text{ X } (5/100)] + [(50/100) \text{ X } (3/100)]}$$

or $P(B_1 \mid A) = 12/77$

$$\text{(ii) } P(B_3 \mid A) = \frac{P(B_3)\, P(A \mid B_3)}{P(B_1)\, P(A \mid B_1) + P(B_2)\, P(A \mid B_2) + P(B_3)\, P(A \mid B_3)}$$

$$\text{or } P(B_1 \mid A) = \frac{(50/100) \text{ X } (3/100)}{[(15/100) \text{ X } (4/100)] + [(35/100) \text{ X } (5/100)] + [(50/100) \text{ X } (3/100)]}$$

or $P(B_3 \mid A) = 30/77$

Index Numbers

Q. 1. Define Index numbers.

Ans. Index numbers are devices for measuring differences in the magnitude of a group of related variables. **- Croxton & Cowden.**

Cost of Living Index

The cost of living index numbers indicate whether the real wages are rising or falling, money wages remaining unchanged.
Methods to construct the cost of living index numbers:

- Simple Aggregate method
- Weighted Aggregate method
- Method of Weighted Relatives

Simple Aggregate method

Cost of living index number = ($\sum P_1/\sum P_0$) X 100
Where:
P_1 = Price of current year
P_0 = Price of base year

Q. 2. Calculate the cost of living index number for the year 1992 taking 1989 as the base year from the following data:

Item	**Price per kg (in Rs.)**	
	In 1989 (P_0)	**In 1992 (P_1)**
A	8	10
B	12	15
C	10	25
D	70	100
E	5	6.5

Solution. $\sum P_0 = 105$, $\sum P_1 = 156.5$
Cost of living index number = (156.5/105) X 100
or Cost of living index number = 149.04

Weighted Aggregate method

Cost of living index number = ($\sum P_1q_0/\sum P_0q_0$) X 100
Where:
P_1 = Price of current year
P_0 = Price of base year
q_0 = Quantity of base year

Q. 3. Calculate the cost of living index number for the year 1992 taking 1989 as the base year from the following data:

Item	**Qty**	**Price per kg (in Rs.)**	
	(Kg)	**In 1989**	**In 1992**
A	30	8	10
B	20	12	15
C	10	10	25

D	2	70	100
E	30	5	6.5

Solution.

Item	Qty (Kg) q_0	Price per kg (in Rs) In 1989 (p_0)	In 1992 (p_1)	p_0 X q_0	p_1 X q_0
A	30	8	10	240	300
B	20	12	15	240	300
C	10	10	25	100	250
D	2	70	100	140	200
E	30	5	6.5	150	195
Total				$\sum P_0q_0 = 870$	$\sum P_1q_0 = 1245$

Cost of living index number = (1245/870) X 100
or Cost of living index number = 143.10

Method of Weighted Relatives

Cost of living index number = $\sum$ PW/$\sum$ W
Where:
P = Price Relative
W = Value Weights, i.e. p_0 X q_0

Q. 4. We are interested in measuring how the prices of certain commodities have changed during previous years:(Dec. 2000, June 2000)

Commodity	Units of purchase	Typical annual consumption	Price 1995	2000
Pen	12 piece	4	60	75
Board Pen	6 piece	1/2	72	92
Registers	1 box	1	950	1100
Aspirin	1 pack	6	59	75
Total			**1141**	**1342**

Calculate index numbers for year 2000 by taking 1995 as base year by Weighted Average of Relatives Indexes (or Method of Weighted Relatives).

Solution.

Commodity	p_0	q_0	p_1	p_0 X q_0	[(p_1/p_0) X 100]	P X W

	(Rs)	(Kg)	(Rs)	(W)	(P)	
Pen	60	4	75	240	125	30000
Board Pen	72	1/2	92	36	127.8	4600.8
Registers	950	1	1100	950	115.8	110010
Aspirin	59	6	75	354	127.1	44993.4
Total	**1141**		**1342**	**∑ W = 1580**		**∑ PW = 189604.2**

Cost of living index number = ∑ PW/∑ W
or Cost of living index number = 189604.2/1580 = 120
Therefore, cost of living index number for the year 2000 is 120.

Chain Base Index Numbers

Calculating Chain Base Index Numbers

Finding out link relative. It is the ratio of current year's price relative to that of previous year's.

Link relative = [(Current Year's Price relative)/(Previous year's price relative)] X 100
and Price relative = (Current year's price/Base year's Price) X 100

Obtaining Chain Base Index: Chain index of any year is the average link relative of that year multiplied by the chain index of the previous year divided by 100.
Chain index for the current year = [(Average link relative of the current year) X (Chain index of the previous year)]/100

Q. 5. The following table gives the average wholesale prices of the four grains for the years 1997 to 2001. Compute chain base index number. (June 2002)

Grain	**1997**	**1998**	**1999**	**2000**	**2001**
Wheat	400	440	360	480	500
Gram	800	880	960	1000	1200
Barley	480	520	420	560	600

Rice	600	640	720	680	720

Solution.

Link relatives based on preceding years

Grain	**1997**	**1998**	**1999**	**2000**	**2001**
Wheat	[(400/400) X 100] = 100	[(440/400) X 100] = 110	[(360/440) X 100] = 81	[(480/360) X 100] = 133	[(500/480) X 100] = 104
Gram	[(800/800) X 100] = 100	[(880/800) X 100] = 110	[(960/880) X 100] = 109	[(1000/960) X 100] = 104	[(1200/1000) X 100] = 120
Barley	[(480/480) X 100] = 100	[(520/480) X 100] = 108	[(420/520) X 100] = 80	[(560/420) X 100] = 133	[(600/560) X 100] = 107
Rice	[(600/600) X 100] = 100	[(640/600) X 100] = 106	[(720/640) X 100] = 112	[(680/720) X 100] = 94	[(720/680) X 100] = 105
Total of link relatives	400	434	382	464	436
Average of Link Relative	400/4 = 100	434/4 = 108.5	382/4 = 95.5	464/4 = 116	436/4 = 109
Chain Base Index Number	100	[(108.5 X 100)/100] = 108.5	[(95.5 X 108.5)/100] = 103.61	[(116 X 103.61)/100] = 120.18	[(109 X 120.18)/100] = 130.99

Q. 6. The following table gives the average wholesale prices of the four grains for the years 1998 to 2001. Compute chain base index number. (Dec. 2001)

Grain	**1998**	**1999**	**2000**	**2001**
Rice	12	18	24	12
Wheat	18	36	54	24
Gram	12	36	60	24
Barley	15	21	54	33

Solution.

Link relatives based on preceding years

Grain	**1998**	**1999**	**2000**	**2001**
Rice	100	150	133	50
Wheat	100	200	150	44
Gram	100	300	166	40
Barley	100	140	257	61

Total of link relatives	400	790	706	195
Average of Link Relative	100	197.5	176.5	48.75
Chain Base Index Number	100	197.5	348.58	169.33

Q. 7. The following table gives the average wholesale prices of the four grains for the years 1996 to 2000. Compute chain base index number. (June 2001)

Grain	**1996**	**1997**	**1998**	**1999**	**2000**
Wheat	100	120	115	125	150
Gram	100	95	105	115	98
Barley	100	110	105	95	120
Rice	100	115	110	120	115

Solution.

	Link relatives based on preceding years				
Grain	**1996**	**1997**	**1998**	**1999**	**2000**
Wheat	100	120	95	108	120
Gram	100	95	110	109	85
Barley	100	110	95	90	126
Rice	100	115	95	109	95
Total of link relatives	400	440	395	416	426
Average of Link Relative	100	110	98.75	104	106.5
Chain Base Index Number	100	110	108.62	112.96	120.30

Q. 8. The following table gives the average wholesale prices of the four fruits for the years 1996 to 2000. Compute chain base index number. (Dec. 2002)

Fruit	**1996**	**1997**	**1998**	**1999**	**2000**
Apple	1600	1760	1440	1920	2000
Orange	3200	3520	3840	4000	4800
Banana	1920	2080	1680	2240	2400
Grapes	2400	2560	2880	2720	2880

Solution.

	Link relatives based on preceding years				
Fruit	**1996**	**1997**	**1998**	**1999**	**2000**
Apple	[(1600/1600) X	[(1760/1600) X	[(1440/1760) X	[(1920/1440) X	[(2000/1920) X

	100] = 100	100] = 110	100] = 82	100] = 133	100] = 104
Orange	[(3200/3200) X 100] = 100	[(3520/3200) X 100] = 110	[(3840/3520) X 100] = 109	[(4000/3840) X 100] = 104	[(4800/4000) X 100] = 120
Banana	[(1920/1920) X 100] = 100	[(2080/1920) X 100] = 108	[(1680/2080) X 100] = 81	[(2240/1680) X 100] = 133	[(2400/2240) X 100] = 107
Grapes	[(2400/2400) X 100] = 100	[(2560/2400) X 100] = 107	[(2880/2560) X 100] = 112	[(2720/2880) X 100] = 94	[(2880/2720) X 100] = 106
Total of link relatives	400	435	384	464	437
Average of Link Relative	400/4 = 100	435/4 = 108.75	384/4 = 96	464/4 = 116	437/4 = 109.25
Chain Base Index Number	100	[(108.75 X 100)/100] = 108.75	[(96 X 108.75)/100] = 104.4	[(116 X 104.4)/100] = 121.10	[(109.25 X 121.10)/100] = 132.30

Miscellaneous Examples

Simpson Rule

$$\int_{x_0}^{x_0+nh} ydx = (h/3) \text{ X } \{(y_0 + y_n) + [4 \text{ X } (y_1 + y_3 + ... + y_{n-1})] + [2 \text{ X } (y_2 + y_4 + ... + y_{n-2})]\}$$

OR

$$\int_{x_0}^{x_0+nh} ydx = (h/3) \text{ X } [(y_0 + y_n) + (4 \text{ X ODD}) + (2 \text{ X EVEN})]$$

Q. 1. Compute the approximate value of the integral (June 2002)

$$I = \int_0^2 (1 + x^2)\, dx$$

using Simpson's rule by taking interval size h as one.

Solution.

x	0	1	2
$y = (1 + x^2)$	1	2	5

Here, $h = 1$, $y_0 = 1$, $y_1 = 2$, $y_2 = 5$

$$\int_0^2 y\,dx = (h/3) \times [(y_0 + y_2) + (4 \times y_1)]$$

or $$\int_0^2 y\,dx = (1/3) \times [(1 + 5) + (4 \times 2)]$$

or $$\int_0^2 y\,dx = 14/3$$

Q. 2. Compute the approximate value of the integral (Dec. 2001)

$$I = \int_0^4 (1 + x + x^2)\,dx$$

using Simpson's rule by taking interval size h as 1.

Solution.

x	0	1	2	3	4
$y = (1 + x + x^2)$	1	3	7	13	21

Here, $h = 1$, $y_0 = 1$, $y_1 = 3$, $y_2 = 7$, $y_3 = 13$, $y_4 = 21$

$$\int_0^4 y\,dx = (h/3) \times \{(y_0 + y_4) + [4 \times (y_1 + y_3)] + (2 \times y_2)\}$$

or $$\int_0^4 y\,dx = (1/3) \times \{(1 + 21) + [4 \times (3 + 13)] + (2 \times 7)\}$$

or $$\int_0^4 y\,dx = 100/3$$

Q. 3. Compute the approximate value of the integral (June 2001)

$$I = \int_{-1}^{1} (x + x^2)\, dx$$

using Simpson's rule by taking interval size h as 1.

Solution.

x	-1	0	1
$y = (x + x^2)$	0	0	2

Here, h = 1, $y_0 = 0$, $y_1 = 0$, $y_2 = 2$

$$\int_{-1}^{1} y\, dx = (1/3) \times [(0 + 2) + (4 \times 0)]$$

or $$\int_{-1}^{1} y\, dx = 2/3$$

Q. 4. Compute the approximate value of the integral (Dec. 2000)

$$I = \int_{0}^{6} [1/(1 + x^2)]\, dx$$

using Simpson's rule by taking interval size h as 1.

Solution.

x	0	1	2	3	4	5	6
$y = [1/(1 + x^2)]$	1	0.5	0.2	0.1	0.058	0.038	0.027

$$\int_{0}^{6} y\, dx = (1/3) \times \{(1 + 0.027) + [4 \times (0.5 + 0.1 + 0.038)] + [2 \times (0.2 + 0.058)]\}$$

or $$\int_{0}^{6} y\, dx = 1.36$$

Q. 5. Compute the approximate value of the integral (Jan. 2001)

$$I = \int_1^3 (1 + x^3)\, dx$$

using Simpson's rule by taking interval size h as 1.

Solution.

x	1	2	3
y = (1 + x³)	2	9	28

$$\int_1^3 y\, dx = (1/3) \text{ X } [(2 + 28) + (4 \text{ X } 9)]$$

or $$\int_1^3 y\, dx = 22$$

Q. 6. Compute the approximate value of the integral (June 2000)

$$I = \int_1^3 (3x^2 + 2x)\, dx$$

using Simpson's rule by taking interval size h as 1.

Solution.

x	1	2	3
y = (3x² + 2x)	5	16	33

$$\int_1^3 y\, dx = (1/3) \text{ X } [(5 + 33) + (4 \text{ X } 16)]$$

or $$\int_1^3 y\, dx = 34$$

Trapezoidal Rule

$$\int_{x_0}^{x_0+nh} y\,dx = (h/2) \times \{(y_0 + y_n) + [2 \times (y_1 + y_2 + y_3 + \ldots + y_{n-1})]\}$$

Q. 7. Evaluate

a) $\int_{1}^{1.3} x^2\,dx$ (Dec. 99)

b) $I = \int_{1}^{3} (1 + x^2 + x^3)\,dx$ (Dec. 2002)

using either Simpson's rule or Trapezoidal rule.

Solution. (a)

Using Trapezoidal rule

x	1	1.05	1.10	1.15	1.20	1.25	1.30
y = x²	1	1.1025	1.21	1.3225	1.44	1.5625	1.69

Here, h = 0.05, $y_0 = 1$, $y_1 = 1.1025$, $y_2 = 1.21$, $y_3 = 1.3225$, $y_4 = 1.44$, $y_5 = 1.5625$, $y_6 = 1.69$

$$\int_{1}^{1.3} y\,dx = (h/2) \times \{(y_0 + y_6) + [2 \times (y_1 + y_2 + y_3 + y_4 + y_5)]\}$$

or $\int_{1}^{1.3} y\,dx = (0.05/2) \times \{(1 + 1.69) + [2 \times (1.1025 + 1.21 + 1.3225 + 1.44 + 1.5625)]\}$

or $\int_{1}^{1.3} y\,dx = 0.3991$

(b) Using Trapezoidal rule

x	1	2	3
y = (1 + x² + x³)	3	13	37

Here, h = 1, $y_0 = 3$, $y_1 = 13$, $y_2 = 37$

$$\int_1^3 y\,dx = (h/2) \text{ X } [(y_0 + y_2) + (2 \text{ X } y_1)]$$

$$\int_1^3 y\,dx = (1/2) \text{ X } [(3 + 37) + (2 \text{ X } 13)]$$

or $\int_1^3 y\,dx = 33$

Q. 8. Compute the approximate value of the integral

$$I = \int_0^2 (x + x^2)\,dx$$

using Trapezoidal rule by taking interval size h as one.

Solution.

x	0	1	2
$y = (x + x^2)$	0	2	6

$$\int_0^2 y\,dx = (1/2) \text{ X } [(0 + 6) + (2 \text{ X } 2)]$$

or $\int_0^2 y\,dx = 5$

Lagrange's Formula

$$f(x) = \frac{(x - x_1)(x - x_2)....(x - x_n)}{(x_0 - x_1)(x_0 - x_2).....(x_0 - x_n)} f(x_0) + \frac{(x - x_0)(x - x_2)....(x - x_n)}{(x_1 - x_0)(x_1 - x_2).....(x_1 - x_n)} f(x_1) + \text{...........} \frac{(x - x_0)(x - x_1)....(x - x_{n-1})}{} f(x_n)$$

$$(x_n - x_0)(x_n - x_1)......(x_n - x_{n-1})$$

Newton's Divided Difference Formula

$f(x) = f(x_0) + (x - x_0)\Delta f(x_0) + (x - x_0)(x - x_1)\Delta^2 f(x_0) + (x - x_0)(x - x_1)(x - x_2)\Delta^3 f(x_0) +$ ....

Q. 9. A table of x versus f(x) is given below. Using any one of the interpolation formulae (Lagrange's/Newton's etc.), find the value of f(x) at x = 4 : (June 1999)

x	1.5	3	6
f(x)	-0.25	2	20

Solution. Given, $x = 4$, $x_0 = 1.5$, $x_1 = 3$, $x_2 = 6$, $f(x_0) = -0.25$, $f(x_1) = 2$, $f(x_2) = 20$

Using Lagrange's Formula

$$f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

$$f(4) = \frac{(4 - 3)(4 - 6)}{(1.5 - 3)(1.5 - 6)} \text{ X } (-0.25) + \frac{(4 - 1.5)(4 - 6)}{(3 - 1.5)(3 - 6)} \text{ X } (2) + \frac{(4 - 1.5)(4 - 3)}{(6 - 1.5)(6 - 3)} \text{ X } (20)$$

or $f(4) = 6$ (approx.)

Using Newton's Divided Difference Formula

The difference table is shown below:

x	**f(x)**	**Δf(x)**	**Δ²f(x)**
1.5	-0.25	[2 - (-0.25)]/(3 - 1.5) = 1.5	
3	2		(6 - 1.5)/(6 - 1.5) = 1
6	20	(20 - 2)/(6 - 3) = 6	

Here, $\Delta f(x_0) = 1.5$, $\Delta^2 f(x_0) = 1$

$f(x) = f(x_0) + (x - x_0)\Delta f(x_0) + (x - x_0)(x - x_1)\Delta^2 f(x_0)$
$f(4) = (-0.25) + [(4 - 1.5) \text{ X } 1.5] + [(4 - 1.5) \text{ X } (4 - 3) \text{ X } 1]$
or $f(4) = 6$

Q. 10. Using any one of the interpolation formulae (e.g., Lagrange's/Newton's etc.) and the following table, find the value of f(x) at x = 3.5 : (Dec. 2000)

x	1	2	5	6
f(x)	1.2	4.8	26	39

Solution. Given, $x = 3.5$, $x_0 = 1$, $x_1 = 2$, $x_2 = 5$, $x_3 = 6$, $f(x_0) = 1.2$, $f(x_1) = 4.8$, $f(x_2) = 26$, $f(x_3) = 39$

Using Lagrange's Formula

$$f(3.5) = \frac{(3.5 - 2)(3.5 - 5)(3.5 - 6)}{(1 - 2)(1 - 5)(1 - 6)} \text{ X } (1.2) + \frac{(3.5 - 1)(3.5 - 5)(3.5 - 6)}{(2 - 1)(2 - 5)(2 - 6)} \text{ X } (4.8) +$$

$$\frac{(3.5 - 1)(3.5 - 2)(3.5 - 6)}{(5 - 1)(5 - 2)(5 - 6)} \text{ X } (26) + \frac{(3.5 - 1)(3.5 - 2)(3.5 - 5)}{(6 - 1)(6 - 2)(6 - 5)} \text{ X } (39)$$

or f(3.5) = 12.7563

Q. 11. The following values of the function f(x) for value of x are given :
f(1) = 4, f(2) = 5, f(7) = 5, f(8) = 4
Using Newton's Divided Difference Formula or Lagrange's Formula, find the value of f(6). (Dec. 99)

Solution. Given, $x = 6$, $x_0 = 1$, $x_1 = 2$, $x_2 = 7$, $x_3 = 8$, $f(x_0) = 4$, $f(x_1) = 5$, $f(x_2) = 5$, $f(x_3) = 4$

Using Newton's Divided Difference Formula

The difference table is shown below:

x	**f(x)**	**Δf(x)**	**Δ^2f(x)**	**Δ^3f(x)**
1	4			
		(5 - 4)/(2 - 1) = 1		
2	5		(0 - 1)/(7 - 1) = -1/6	
		(5 - 5)/(7 - 2) = 0		0
7	5		(-1 - 0)/(8 - 2) = -1/6	
		(4 - 5)/(8 - 7) = -1		
8	4			

Here, $\Delta f(x_0) = 1$, $\Delta^2 f(x_0) = -1/6$, $\Delta^3 f(x_0) = 0$

$f(x) = f(x_0) + (x - x_0)\Delta f(x_0) + (x - x_0)(x - x_1)\Delta^2 f(x_0) + (x - x_0)(x - x_1)(x - x_2)\Delta^3 f(x_0)$
$f(6) = 4 + [(6 - 1) X 1] + [(6 - 1) X (6 - 2) X (-1/6)] + [(6 - 1) X (6 - 2) X (6 - 7) X 0]$
or $f(6) = 5.66$

Bessel's Formula

(Derivatives at mid or near mid)

$$y_u = \left[\frac{(y_0 + y_1)}{2}\right] + \left[\frac{(2u - 1)}{2} \text{ €€ X } (\Delta y_0)\right] + \left[\frac{[u(u - 1)]}{2!} X \frac{(\Delta^2 y_{-1} + \Delta^2 y_0)}{2}\right] +$$

$$\left[\frac{\{u[u - (1/2)](u - 1)\}}{3!} \text{ €€ X } (\Delta^3 y_{-1})\right] + \left[\frac{[u(u^2 - 1)(u - 2)]}{4!} X \frac{(\Delta^4 y_{-2} + \Delta^4 y_{-1})}{2}\right]$$

$$+ \left[\frac{\{u[u - (1/2)](u^2 - 1)(u - 2)\}}{5!} \text{ €€ X } (\Delta^5 y_{-2})\right] + \left[\frac{[u(u^2 - 1)(u^2 - 2)(u - 3)]}{6!} X \frac{(\Delta^6 y_{-3} + \Delta^6 y_{-2})}{2}\right] + ..$$

Where $u = (x - a)/h$
Therefore, $du/dx = 1/h$

$(d/dx) [y(u)] = \{(d/du)(du/dx) [y(u)]\} = \{(d/du)[y(u)](1/h)\} = [y'(u)]/h$

Differentiating w.r.t. u

$$y'_{(u)} = \Delta y_0 + \left[\frac{(2u - 1)}{2!} X \frac{(\Delta^2 y_{-1} + \Delta^2 y_0)}{2}\right] + \left[\frac{[3u^2 - 3u + (1/2)]}{3!} X (\Delta^3 y_{-1})\right] +$$

$$\left[\frac{(4u^3 - 6u^2 - 2u + 2)}{4!} X \frac{(\Delta^4 y_{-2} + \Delta^4 y_{-1})}{2}\right] +....$$

Q. 12. A portion of a table of sines is given below: (June 2002, June 99)

Angle in Radians	**Sine**
0.25	0.2474

0.26	0.2571
0.27	0.2667
0.28	0.2764
0.29	0.2860

Find the derivative of this function at x = 0.27.

Solution. Here, we have to find out the derivative near the middle of the table. So, we use **Bessel's formula.**
The difference table is shown below:

u	**x**	**f(x)**	**Δ**	**Δ^2**	**Δ^3**	**Δ^4**
-2	0.25	0.2474				
			0.0097			
-1	0.26	0.2571		-0.0001		
			0.0096		-0.0002	
0	0.27	0.2667		0.0001		0
			0.0097		-0.0002	
1	0.28	0.2764		-0.0001		
			0.0096			
2	0.29	0.2860				

a = 0.27, h = 0.01
u = (x - 0.27)/h
Therefore, du/dx = 1/h
(d/dx) [y(u)] = [y'(u)]/h .....(i)

$$y'_{(u)} = \Delta y_0 + \left[\frac{(2u - 1)}{2!} \text{ X } \frac{(\Delta^2 y_{-1} + \Delta^2 y_0)}{2}\right] + \left[\frac{[3u^2 - 3u + (1/2)]}{3!} \text{ X } (\Delta^3 y_{-1})\right] + \left[\frac{(4u^3 - 6u^2 - 2u + 2)}{4!} \text{ X } \frac{(\Delta^4 y_{-2} + \Delta^4 y_{-1})}{2}\right]$$

Putting u = 0
$y'_{(0)} = \Delta y_0 - [(1/4) \text{ X } (\Delta^2 y_{-1} + \Delta^2 y_0)] + [(1/12) \text{ X } (\Delta^3 y_{-1})] + [(1/24) \text{ X } (\Delta^4 y_{-2} + \Delta^4 y_{-1})]$
or $y'_{(0)} = 0.0097 - \{(1/4) \text{ X } [0.0001 + (-0.0001)]\} + [(1/12) \text{ X } (-0.0002)] + [(1/24) \text{ X } 0]$
or $y'_{(0)} = 0.0096$

From equation (i)
(d/dx) [y(u)] = [y'(0)]/h
or $y'_{(0.27)} = (0.0096)/0.01$
or $y'_{(0.27)} = 0.96$

Q. 13. A portion of a table of a function f(x) is given below: (June 2000)

x	0.71	0.72	0.73	0.74	0.75	0.76
f(x)	0.2474	0.2571	0.2667	0.2764	0.2860	0.2910

Find the derivative of this function at x = 0.74.

Solution. The difference table is shown below:

u	**x**	**f(x)**	**Δ**	**Δ^2**	**Δ^3**	**Δ^4**	**$\Delta 5$**
-3	0.71	0.2474	0.0097				
-2	0.72	0.2571	0.0096	-0.0001	0.0002		
-1	0.73	0.2667	0.0097	0.0001	-0.0002	-0.0004	-0.0039
0	0.74	0.2764	0.0096	-0.0001	-0.0045	-0.0043	
1	0.75	0.2860	0.0050	-0.0046			
2	0.76	0.2910					

Differentiating w.r.t. u and putting u = 0, we get
$y'_{(0)} = \Delta y_0$ - [(1/4) X ($\Delta^2 y_{-1} + \Delta^2 y_0$)] + [(1/12) X ($\Delta^3 y_{-1}$)] + [(1/24) X ($\Delta^4 y_{-2} + \Delta^4 y_{-1}$)] - [(1/120) X ($\Delta^5 y_{-2}$)]
or $y'_{(0)}$ = 0.0096 - {(1/4) X [(-0.0001) + (-0.0046)]} + [(1/12) X (-0.0045)] + [(1/24) X (-0.0043)] - [(1/120) X (-0.0039)]
or $y'_{(0)}$ = 0.0103

Here, h = 0.01
Therefore, $y'_{(0.74)}$ = (0.0103)/0.01 = 1.03

FORTRAN

FORTRAN Expressions

Q. 1. Write a FORTRAN statement for each of the following mathematical expressions:

i. **$z = e^{x+y} + \log(x + y^2)$** (June 2002)
ii. **$z = [(ab)/(c + d)]^2$** (June 2002)

iii. $z = \sqrt{\quad}$ **[sin (x + y^2) + tan^2 xy]** (June 2002)
iv. **a [(x + y)/z]$^{3.5}$** (Dec. 2001)

v. **| sin X | + log $\sqrt{\quad}$ (3X^2 + 5Y^2)** (Dec. 2001)
vi. **X = [(ab)/(c + d^k/m + k)] + a** (June 2001)
vii. **u = e$^{| x2 - y2 |}$** (June 2001)

viii.

(viii) v = A$^{B^C}$ (June 2001)

ix. **x = a$^{z + 1}$** (Dec. 2000)
x. **x = | y - b |** (Dec. 2000)

xi. **x = $\sqrt{\quad}$ (y + b)** (Dec. 2000)
xii. **x = e$^{-y2/2}$ or x = 1/e$^{y2/2}$** (Dec. 2000)
xiii. **u = (log (x + y) - tan (x + ny))^2** (Jan. 2001)
xiv. **v = e^{xy} - | x^2 - y^2 |** (Jan. 2001)

xv. **w = $\sqrt{\quad}$ ((x)^y)^z** (Jan. 2001)
xvi. **z = tan (3x - 2y) + 4e^{xy}** (June 2000)

xvii. **z = | x^2 - y^2 | + $\sqrt{\quad}$ (5X^2 + 8Y^2)** (June 2000)
xviii. **z = [(p + q)/(r + s)]^3** (June 2000)
xix. **z = e$^{x + y}$ - sin (x + ny)** (Dec. 99)
xx. **z = [(a + b)/(c + d)]^3** (Dec. 99)

xxi. **z =** $\sqrt{(5X^2 + 8Y^2)}$ (Dec. 99)
xxii. **z = $\cot^2 (x^2)$ - log x.y (Dec. 2002)**
xxiii. **z = $| x^2 - y^2 | + 6e^{x.y}$ (Dec. 2002)**
xxiv. **z =** $\sqrt{\sin^2 (xy) + ((x)^y)^z}$ **(Dec. 2002)**

Ans.

i. Z = EXP (X + Y) + LOG (X + Y * * 2)
ii. Z = ((A * B)/(C + D)) * * 2
iii. Z = SQRT ((SIN (X + Y * * 2)) + (TAN * * 2 (X * Y)))
iv. (A * (X + Y)/Z) * * 3.5
v. ABS (SIN (X)) + LOG (SQRT (3 * X * * 2 + 5 * Y * * 2))
vi. X = ((A * B)/(C + D * * K/M + K)) + A
vii. U = EXP (ABS (X * * 2 - Y * * 2))
viii. V = A * * (B * * C)
ix. (ix) X = A * * (Z + 1)
x. X = ABS (Y - B)
xi. X = SQRT (Y + B)
xii. X = 1/(EXP (Y * * 2)/2)
xiii. U = (LOG (X + Y) - TAN (X + N * Y)) * * 2
xiv. V = EXP (X * Y) - ABS (X * * 2 - Y * * 2)
xv. W = SQRT ((X * * Y) * * Z)
xvi. Z = TAN (3 * X - 2 * Y) + 4 * EXP (X * Y)
xvii. Z = ABS (X * * 2 - Y * * 2) + SQRT (5 * X * * 2 + 8 * Y * * 2)
xviii. Z = ((P + Q)/(R + S)) * * 3
xix. Z = EXP (X + Y) - SIN (X + N * Y)
xx. Z = ((A + B)/(C + D)) * * 3
xxi. Z = SQRT (5 * X * * 2 + 8 * Y * * 2)
xxii. Z = (COT * * 2 (X * * 2)) - LOG (X * Y)
xxiii. Z = ABS (X * * 2 - Y * * 2) + (6 * EXP (X * * Y))
xxiv. Z = SQRT ((SIN * * 2 (X * Y)) + ((X * * Y) * * Z))

Q. 2. (a) Suppose integer variables P, Q and R contain 8, 16 and 24 respectively. Find the value of each of the following logical expressions: (June 2002)
(i) Q .EQ. 8 .AND. .NOT. P .LT. R - 12
(ii) .NOT. (P .GT. Q .OR. R .LT. 7)

(b) Suppose at some stage values of integer variables P, Q and R are respectively 2, 3 and 4. Find the value of the variable S after execution of each pair of statements given below: (Dec. 2002)
(i) P = -P + P * R * * 2 + Q
S = -P + P * R * * 2 + Q
(ii) P = ABS (P - R * Q)/5
S = ABS (P - R * Q)/5

(c) Suppose the integer variables X and Y contain respectively 2 and 7. Find the final values of X and Y in each of the following cases: (Dec. 2002)
(i) If (X .LE. Y) X = X + 5
X = X + 3
(ii) IF (X .EQ. Y + 2) GOTO 15
X = X + 3
15 X = X + Y

Solution. a) Given, P = 8, Q = 16 and R = 24
(i) Q .EQ. 8 .AND. .NOT. P .LT. R - 12
= 16 .EQ. 8 .AND. .NOT. 8 .LT. 24 - 12
= 16 .EQ. 8 .AND. .NOT. 8 .LT. 12
= F .AND. .NOT. T
= F .AND. F
= F

(ii) .NOT. (P .GT. Q .OR. R .LT. 7)
= .NOT. (8 .GT. 16 .OR. 24 .LT. 7)
= .NOT. (F .OR. F)
= .NOT. (F)
= T

b) (i) P = -P + P * R * * 2 + Q
or P = -2 + 2 * 4 * * 2 + 3
or P = -2 + 2 X $(4)^2$ + 3
or P = 33
S = -P + P * R * * 2 + Q
or S = -33 + 33 * 4 * * 2 + 3
or S = 498

(ii) P = ABS (P - R * Q)/5
or P = ABS (2 - 4 * 3)/5
or P = | 2 - 4 X 3 |/5
or P = | -10 |/5
or P = 10/5

or P = 2
S = ABS (P - R * Q)/5
or S = ABS (2 - 4 * 3)/5
or S = 2

c) (i) The condition is true as X < Y.
X will be incremented by 5 and becomes 7.
Again, X will be incremented by 3 and becomes 10.

(ii) The condition is false as X = 2 and Y + 2 = 9.
X = 2 + 3 = 5
Then X = 5 + 7
Therefore, the final values of X and Y are 12 and 7 respectively.

Q. 3. Suppose the variables A, B and C respectively contain the values 3, 4 and 5. Find the value of each of the following logical expressions: (Dec. 2001)
(i) (A + C) .EQ. 2 * B .AND. 2 * (C - A) .EQ. B
(ii) .NOT. (3 .EQ. C - 2 .AND. A .LE. C)
(iii) .NOT. C .GT. A .OR. B .LT. 5

Solution. Given, A = 3, B = 4 and C = 5
(i) (A + C) .EQ. 2 * B .AND. 2 * (C - A) .EQ. B
= (3 + 5) .EQ. 2 * 4 .AND. 2 * (5 - 3) .EQ. 4
= 8 .EQ. 8 .AND. 4 .EQ. 4
= T .AND. T
= T

(ii) .NOT. (3 .EQ. C - 2 .AND. A .LE. C)
= .NOT. (3 .EQ. 5 - 2 .AND. 3 .LE. 5)
= .NOT. (3 .EQ. 3 .AND. 3 .LE. 5)
= .NOT. (T .AND. T)
= .NOT. (T)
= F

(iii) .NOT. C .GT. A .OR. B .LT. 5
= .NOT. 5 .GT. 3 .OR. 4 .LT. 5
= .NOT. F .OR. T
= T .OR. T
= T

Q. 4. Suppose J, K and L contain 10, 20 and 30 respectively. Find the value of each of the following logical expressions: (June 2001)

(i) .NOT. (5 .EQ. J - 5 .AND. 2 * K .EQ. J + L)
(ii) 2 * J .EQ. K .AND. K .LE. L

Solution. Given, J = 10, K = 20 and L = 30
(i) .NOT. (5 .EQ. J - 5 .AND. 2 * K .EQ. J + L)
= .NOT. (5 .EQ. 10 - 5 .AND. 2 * 20 .EQ. 10 + 30)
= .NOT. (5 .EQ. 5 .AND. 40 .EQ. 40)
= .NOT. (T .AND. T)
= .NOT. (T)
= F

(ii) 2 * J .EQ. K .AND. K .LE. L
= 2 * 10 .EQ. 20 .AND. 20 .LE. 30
= 20 .EQ. 20 .AND. 20 .LE. 30
= T .AND. T
= T

Q. 5. Find the value of each of the following FORTRAN expressions: (Dec. 2000)
(i) A * * B * * C * D
Where A, B, C and D are integers with values A = 2, B = 3, C = 2, D = 2
(ii) 17 + 3 .LT. 12 .OR. 7 .GT. 9
(iii) '332' // '+148'

Solution.
(i) A * * B * * C * D
= 2 * * 3 * * 2 * 2
= (2^9) X 2
= 512 X 2
= 1024

(ii) 17 + 3 .LT. 12 .OR. 7 .GT. 9
= 20 .LT. 12 .OR. 7 .GT. 9
= F .OR. F
= F

(iii) '332' // '+148'
= 332 + 148
= 480

Q. 6. Suppose J, K and L contain 10, 20 and 30 respectively. Find the value of each of the following logical expressions: (Jan. 2001)
(i) .NOT. J .GT. K .OR. L .LT. 5
(ii) K .EQ. 10 .AND. .NOT. J .LT. L - 15

Solution. Given, J = 10, K = 20 and L = 30
(i) .NOT. J .GT. K .OR. L .LT. 5
= .NOT. 10 .GT. 20 .OR. 30 .LT. 5
= .NOT. F .OR. F
= T .OR. F
= T

(ii) K .EQ. 10 .AND. .NOT. J .LT. L - 15
= 20 .EQ. 10 .AND. .NOT. 10 .LT. 30 - 15
= F .AND. .NOT. T
= F .AND. F
= F

Q. 7. Suppose P, Q and R contain 5, 10 and 15 respectively. Find the value of each of the following logical expressions: (June 2000)
(i) 2 * P .EQ. Q .AND. Q .LE. R
(ii) .NOT. (5 .EQ. P - 2 .AND. Q .LE. R)
(iii) .NOT. P .GT. Q .OR. R .LT. 5

Solution. Given, P = 5, Q = 10 and R = 15
(i) 2 * P .EQ. Q .AND. Q .LE. R
= 2 * 5 .EQ. 10 .AND. 10 .LE. 15
= 10 .EQ. 10 .AND. 10 .LE. 15
= T .AND. T
= T

(ii) .NOT. (5 .EQ. P - 2 .AND. Q .LE. R)
= .NOT. (5 .EQ. 5 - 2 .AND. 10 .LE. 15)
= .NOT. (5 .EQ. 3 .AND. 10 .LE. 15)
= .NOT. (F .AND. T)
= .NOT. (F)
= T

(iii) .NOT. P .GT. Q .OR. R .LT. 5
= .NOT. 5 .GT. 10 .OR. 15 .LT. 5
= .NOT. F .OR. F
= T .OR. F
= T

Q. 8. Suppose at some values of variables A, B and C are 2, 3 and 4 respectively. Find the value of L after each pair of statements: (Dec. 99)
(i) A = A + A * C * * 2 + B
L = A + A * C * * 2 + B

(ii) A = ABS (A - C * B)/5
L = ABS (A - C * B)/5

Solution.
(i) A = A + A * C * * 2 + B
or A = 2 + 2 * 4 * * 2 + 3
or $A = 2 + 2 \times (4)^2 + 3$
or A = 2 + 32 + 3
or A = 37
L = A + A * C * * 2 + B
or L = 37 + 37 * 4 * * 2 + 3
or L = 632

(ii) A = ABS (A - C * B)/5
or A = ABS (2 - 4 * 3)/5
or A = | -10 |/5
or A = 10/5
or A = 2
L = ABS (A - C * B)/5
or L = ABS (2 - 4 * 3)/5
or L = 10/5
or L = 2

Q. 9. Write each of the following statements in FORTRAN: (Dec. 2001)
(i) If R = 2S + T, go to statement labelled 87
(ii) If S ≠ 11, go to statement labelled 44
(iii) If 3S > 4T, stop

Ans. (i) IF (R .EQ. (2 * S + T)) GOTO 87
Ans. (ii) IF (S .NE. 11) GOTO 44
Ans. (iii) IF (3 * S .GT. 4 * T) STOP

Q. 10. For J = 2 and K = 5, find the final values of J and K after each program segment: (June 2001)
(i) If (J - K) 10, 10, 20
10 J = K
20 J = J + 2
(ii) IF (J .GE. K) J = K + 2
J = J + 2

Ans. (i) J = 5 and K = 5
Ans. (ii) J = 4 and K = 5

Q. 11. Suppose J and K contain respectively 3 and 5. Find the final values of J and K after each program segment: (Jan. 2001)
If (J .EQ. K) J = J + 2
J = J + 2
IF (J .GE. K + 1) GOTO 10
J = J + 2
10 J = J + K

Solution.
Since J is not equal to k, it will remain unchanged.
J will be incremented by 2 and becomes 5.
The condition is false as J = 5 and K + 1 = 6.
J will be incremented by 2 and becomes 7.
J will be incremented by the value of K(5) and becomes 12.
Therefore, the final values of J and K are 12 and 5 respectively.

Q. 12. Write the following statements in FORTRAN: (June 2000)
(i) If P = 2Q + 3, go to statement labelled 45
(ii) If 2P > 3Q, stop
(iii) If J ≠ 7, go to statement labelled 32

Ans. (i) IF (P .EQ. (2 * Q + 3)) GOTO 45
Ans. (ii) IF (2 * P .GT. 3 * Q) STOP
Ans. (iii) IF (J .NE. 7) GOTO 32

FORTRAN Constants

1. Character constant

A character constant is a character string enclosed in apostrophes (sometimes called "single quotes" when used this way).

2. Real constant

A string of digits and one period, used as a decimal point, is called a real constant. The following are real constants:
13.6 .1234567 3.0 00.30

Real constants are also written with an exponent (E). The letter "E" is read as "times ten to the power" and the integer following the "E" is a power of ten to be multiplied by the real number. This form, called the exponential notation, is useful for writing very large or very small numbers. For instance, 2.3E5 is 2.3 times ten to the power 5 or 2.3 X 100000

= 230000. The integer power may have a minus sign preceding it as in the real constant 2.3E-5, which is 2.3 times 10 to the power -5 or 2.3 X .00001 = .000023.

3. Integer constant

An integer constant is a string consisting only of the digits 0 to 9, such as the following:
24 0 1234567

Q. 13. Write which of the following FORTRAN constants are invalid:
(i) -3/4 (June 2002, June 2001)
(ii) 0.01540E05 (Dec. 2001)
(iii) -0.148E - 5 (Dec. 2001)
(iv) 125.8E (Dec. 2001)
(v) 12.5E + 4 (June 2001)
(vi) 12345 (June 2001)
(vii) 25 (June 2001)
(viii) 30, 416 (Jan. 2001)
(ix) 1.2E14 (Jan. 2001)
(x) -1/2 (Jan. 2001)
(xi) 4E1.4 (June 2000, Dec. 99)
(xii) 97.52 (June 2000)
(xiii) 2, 489.70 (June 2000, Dec. 99)
(xiv) 1.6E128 (Dec. 99)

Ans. (i) Invalid constant because no fractions are allowed.
Ans. (ii) Valid real constant.
Ans. (iii) Valid real constant.
Ans. (iv) Invalid constant.
Ans. (v) Valid real constant.
Ans. (vi) Valid integer constant.
Ans. (vii) Valid integer constant.
Ans. (viii) Invalid constant as comma (,) is not allowed in any constant.
Ans. (ix) Valid constant.
Ans. (x) Invalid constant because no fractions are allowed.
Ans. (xi) Invalid constant because exponent cannot be a real number.
Ans. (xii) Valid integer constant.
Ans. (xiii) Invalid constant because comma (,) is not allowed.
Ans. (xiv) Invalid constant because exponent is too large.

Q. 14. Write which of the following varible names are invalid in FORTRAN?
(i) STOP (June 2001, Dec. 99)
(ii) ROOTZ (June 2001, June 2000, Dec. 99)
(iii) 2RATES (June 2001)

(iv) ROLL2 (Dec. 2001)
(v) PROGRAM (Dec. 2001)
(vi) X + Y (Jan. 2001)
(vii) END (Jan. 2001, June 2000)
(viii) 2ROOT (Jan. 2001)
(ix) Y - 55 (June 2000)
(x) M + 30 (Dec. 99)

Ans. (i) Invalid variable because the keyword of Fortran cannot be used as variable name.
Ans. (ii) Valid variable.
Ans. (iii) Invalid variable because first character must be alphabetic.
Ans. (iv) Valid variable.
Ans. (v) Valid variable.
Ans. (vi) Invalid variable as '+' sign cannot be allowed in the variable name.
Ans. (vii) Invalid variable because the keyword of Fortran cannot be used as variable name.
Ans. (viii) Invalid variable because first character must be alphabetic.
Ans. (ix) Invalid variable as '-' sign cannot be allowed in the variable name.
Ans. (x) Invalid variable as '+' sign cannot be allowed in the variable name.

FORTRAN Statements

1. INPUT – OUTPUT STATEMENTS

List directed input statement:

READ * , V1, V2, ... V_N

All the variables must be separated by commas (,).

List directed output statement:

PRINT *, V1, V2, ... V_N

All the variables must be separated by commas (,).

2. ENDING PROGRAMS STATEMENT

END Statement

The END statement is the last statement in all FORTRAN 77 programs. Its purpose is to inform the compiler about the end of any source program. It should not be labeled. It is non executable and not translated into machine language. Its format is:

END

STOP Statement

The STOP statement instructs the computer to terminate the execution of the object program. Its syntax is:

[1] STOP

Here, label is optional.

3. CONTROL TRANSFER COMMAND

1. GOTO and
2. Computed GOTO

GOTO Command

The GOTO statement is used to branch around one or more statements.

[1] GOTO s

s is the statement number.

Computed GOTO Command

[1] GOTO (s1, s2, ...sn), v

where s1,s2, ..., sn are statement numbers and v is an integer expression. If the value of the integer expression is not one of the integer 1,2, ...n, the statement is not executed, and execution continues with the next statement of the program. If this value is one of the integer 1,2, ...n, say j, control is transferred to the statement whose number is nj.

4. DIMENSION STATEMENT

The name and the range of subscripts of an array may be declared in a DIMENSION statement of the form

DIMENSION list

Where list is a list of array declaration.

The following statements declare a one-dimensional array named NUM.

DIMENSION NUM (3)

DIMENSION NUM (1:3)

The following statements declare a two-dimensional array named NUM.

DIMENSION NUM (3,4)

DIMENSION NUM (1:3,1:4)

Two or more array names could be declared in single statement such as;

DIMENSION NUM (3,4), NUM2(3)

The information regarding the dimension of an array can also be given in the TYPE specification statement.

For example, an array NAME consisting of 10 characters string, each of which has length 25 can be declared by the following statement:

CHARACTER * 25 NAME (10)

OR

DIMENSION NAME(10)

CHARACTER * 25 NAME

5. COMMON STATEMENT

This statement is used to define and introduce global variables in FORTRAN. Two types of COMMON statements are available: Blank and labeled COMMON statement.

Blank COMMON statement

When COMMON statement is used in a program, a separate area is set up in the computer memory. Space in this area is allocated only to the variables appearing with the COMMON statement.

For example: COMMON ALPHA, BETA, GAMA.

The variables ALPHA, BETA, and GAMA are assigned the memory locations #1, #2, and #3 respectively in the global area. If another program unit contains the statement COMMON A, B, C, then these three variables are also assigned the first three memory locations in the common area. The following table shows the relationship:

Variable	**Blank Common location**	**Variable**
ALPHA	1	A
BETA	2	B
GAMA	3	C

Therefore, any reference to ALPHA is equivalent to the reference to A, and so on.

Labeled COMMON statement

The named COMMON allows establishment of more than one common regions each of which is uniquely named. The general form of a labeled COMMON statement is:

COMMON / NAME1/ list1,list2n /NAME2/ list3, list4

Variable list1and list2 are common variable and have been grouped together and given a name NAME1. Variable list3and list4 are common variable and have been grouped together and given a name NAME2.

6. FORMAT statement

The FORMAT statement specifies the form of the data being read or written. If the form of the data remains unchanged, the same format statement can be used for reading and writing. Moreover, the same format statement can be used for more than one input-output statements.

statement number FORMAT(sets of specifications).

Each specification set consists of three elements: **edit descriptor, field size,** and **decimal location.**

The edit descriptor specifies the type of the data.
The field size is the maximum no. of character positions on input or output media.
The decimal location is expressed as the no. of places from the right of the field.

Example:

FORMAT (Iw)

1. FORMAT (I5)

I is the edit descriptor specifying that the data is of integer type and w is the field size/width

FORMAT (Fw.d)

106 FORMAT (F10.2)

F is the edit descriptor specifying that the data is of real type, w is an integer indicating the field size, and d is the decimal location expressed as the no. of places form the right of the field.

Q.15. Locate the error, if any, in each of the following WRITE-FORMAT pairs:
(June 2002)
(i) WRITE (3, 35) X, Y, Z
35 FORMAT (1X, 3E 18.9)
(ii) WRITE (*, 40) A, J, B, C, K
40 FORMAT (F7.1, 8I, E6.2)

Ans. (i) No spaces are allowed in the specification 3E 18.9. So the specification 3E 18.9 must be written as 3E18.9. Format specification of two variables (Y, Z) is not defined.
(ii) The integer data must be in the form of Iw. So 8I must be written as I8. Format specification of two variables (C, K) is not defined.

Q.16. Locate the error, if any, in each of the following WRITE-FORMAT pairs:
(Dec. 99)
(i) WRITE (*, 30), A, B, N
30 FORMAT (F10.2, 3X, I8, 5X, I6)
(ii) WRITE (*, 40) A, J, B, C, K
40 FORMAT (F7.1X, I8, E6.2)

Ans. (i) In WRITE statement, comma is not allowed after right parenthesis.
(ii) Format specification of two variables (C, K) is not defined.

FORTRAN Programs

Q. 17. Write a program that accepts as input a number F that represents temperature in degrees Fahrenheit, converts that number to its equivalent in degrees centigrade, and outputs the converted number.

Solution.

```
REAL F, C
READ *, F
C =(5/9) * (F - 32)
PRINT *, 'DEGREES CENTIGRADE = ' , C
STOP
END
```

Q. 18. Write a program that reads values for the three sides of a triangle, calculates its perimeter and its area, and outputs these values.

Solution.

```
REAL A, B, C
REAL P, S, AREA

READ *, A, B, C

P = A + B + C
S = P/2
AREA = SQRT(S*(S-A)*(S-B)*(S-C))

PRINT *, 'PERIMETER = ', P
PRINT *, 'AREA = ', AREA

STOP
END
```

Q. 19. Write a program that reads a four-digit number and finds the sum of the individual digits.

Solution.

```
INTEGER DIGIT1, DIGIT2, DIGIT3, DIGIT4, SUM

READ *, N
```

```
DIGIT1 = N - (N/10)*10
N = N/10

DIGIT2 = N - (N/10)*10
N = N/10

DIGIT3 = N - (N/10)*10
N = N/10

DIGIT4 = N
SUM = DIGIT1 + DIGIT2 + DIGIT3 + DIGIT4

PRINT *, SUM

STOP
END
```

Q. 20. Write a program to find out the largest of the given three numbers.

Solution.

```
REAL X1, X2, X3
READ *, X1, X2, X3

IF (X1 .GT. X2) THEN
IF (X1 .GT. X3) THEN
PRINT *, X1
ELSE
PRINT *, X3
ENDIF
ELSE
IF (X2 .GT. X3) THEN
PRINT *, X2
ELSE
PRINT *, X3
ENDIF

ENDIF

STOP
END
```

Q. 21. Write a program that builds an item-price database on a direct-access file then uses it to find price of an item, given its item number.

Solution.

```
C     MAIN PROGRAM

COMMON UNT, RECSZ, FNAME

INTEGER UNT, RECSZ
CHARACTER*15 FNAME

UNT = 10
RECSZ = 25
FNAME = ‘PRICE DATABASE’

CALL BLDDAT
CALL USEDAT

STOP
END

SUBROUTINE BLDDAT

COMMON UNT, RECSZ, FNAME

INTEGER UNT, RECSZ
CHARACTER*15 FNAME
CHARACTER STAT*3
INTEGER ITEM
REAL PRICE

STAT = ‘NEW’
CALL FOPEN(STAT)

10    READ ( 5, *, END = 99) ITEM, PRICE
WRITE ( UNT, REC = ITEM) ITEM, PRICE
GO TO 10

99    CLOSE (UNT)
RETURN

END

SUBROUTINE USEDAT

COMMON UNT, RECSZ, FNAME
```

```
INTEGER UNT, RECSZ
CHARACTER*15 FNAME
CHARACTER STAT*3
INTEGER ITEM, ITEMNO
REAL PRICE

STAT = ‘OLD’
CALL FOPEN(STAT)

10    PRINT *, ‘ENTER ITEM NUMBER’
READ * , INEMNO

IF (ITEMNO .LE. 0) GO TO 99

READ( UNT, REC = ITEMNO) ITEM, PRICE
PRINT *, ‘PRICE OF ITEM’, ITEM, ‘IS’, PRICE

GO TO 10

99    CLOSE(UNT)
RETURN

END
```

SUBROUTINE FOPEN(STAT)

```
CHARACTER*3 STAT

COMMON UNT, RECSZ, FNAME

INTEGER UNT, RECSZ
CHARACTER*15 FNAME

OPEN(UNT,IOSTAT = ISTAT, ERR = 99, FILE = FNAME, STATUS = STAT,
1 ACCESS = ’DIRECT’, FORM = ’UNFORMMATTED’, RECL = RECSZ)
RETURN

99    PRINT*, ‘ERROR IN OPENING’, FNAME, ‘;ERR CODE = ‘ ,ISTAT

STOP
END
```

Q. 22. Write a program to find out the largest of the given N numbers.

Solution.

```
DIMENSION X(100)
REAL X, LARGE
INTEGER I, N

C    CAN SELECT FROM MAXIMUM OF 100 NUMBERS

READ *, N
IF (N .GT. 100) THEN
PRINT *, 'TOO MANY NUMBERS'
STOP
ENDIF

C    READ THE NUMBERS

READ *, (X(I), I = 1, N)

C    INITIALIZE

LARGE = X(1)
I = 2

C    PICK THE LARGEST NUMBER.

10 IF (X(I) .GT. LARGE) LARGE = X(I)
I = I + 1
IF (I .LE. N) GOTO 10

C    LARGE HAS THE LARGEST NAUMBER

PRINT *, LARGE
STOP
END
```

Q. 23. Given the number of hours worked and the hourly wage rate. Write a program to compute the gross salary and net pay for the employee, assuming the tax deduction to be at the rate of 20% of the employee's gross salary, if it is less than RS. 1000/-, and at the rate of 30% otherwise.

Solution.

```
CHARACTER *10 NAME
REAL HOURS, RATE
REAL GROSS, NET, TAX, TXRATE

READ *, NAME, HOURS, RATE

GROSS = HOURS * RATE

IF (GROSS .LT. 1000) THEN
TXRATE = 0.2
ELSE
TXRATE = 0.3
ENDIF

TAX = TXRATE * GROSS

NET = GROSS – TAX

PRINT *, NAME, GROSS, TAX, NET

STOP
END
```

Q. 24. Write a Program that reads an integer N and computes N!. (June 99)

Solution.

```
INTEGER NUMBER, VALUE , N

10    READ *, NUMBER

IF (NUMBER .LT. 0) THEN
PRINT *, 'POSITIVE NUMBER PLEASE'
GO TO 10
ENDIF

VALUE = 1

IF (NUMBER .EQ. 0) GO TO 30

DO 20 N = 2, NUMBER
VALUE = VALUE * N
```

```
20    CONTINUE

30    PRINT *, VALUE

STOP
END
```

Q. 25. Write a Program to check whether a given number is prime or not.

Solution.

```
INTEGER NUMBER, TRY

READ *, NUMBER

DO 10 TRY = 2, NUMBER/2

IF( MOD(NUMBER,TRY).EQ.0) THEN
PRINT *, NUMBER,' IS NOT A PRIME'
STOP
ENDIF

10    CONTINUE

PRINT *, NUMBER ,'IS A PRIME' STOP
END
```

Q. 26. X is an integer array of N elements (N = 10). Cyclically permute the values of X so that X_1 contains the original value of X_2, X_2 contains the value X_3 and so on, with X_N containing the original value of X_1.

Solution.

```
PARAMETER (N = 10)

INTEGER X(N)
INTEGER TEMP

READ *, X

C    SAVE THE FIRST ELEMENT

TEMP = X(1)

C    SHIFT LEFT BY 1 ELEMENTS X(2) THROUGH X (N)
```

```
DO 10 I = 2, N
X(I-1) = X(I)
10    CONTINUE
```

C   RESTORE SAVED VALUE OF FIRST ELEMENT IN THE LAST

```
X(N) = TEMP
PRINT *, X

STOP
END
```

Q. 27. Write a program that reads a 2 * 3 matrix column-wise and print it row-wise.

Solution.

```
INTEGER M(2,3)
READ (5,70) M
WRITE (6, 80) ( (M(I,J) ,J-1,3 ,I=1 ,2) 70 FORMAT*2I3)
80 FORMAT(3(1x,I3))

END
```

Q. 28. Write a program that prints the monthly report of activity in a bank account from a set of input data. The first line (card) of input data provides the opening balance. All other lines (cards) contain a transaction code (1= deposit, 2= withdrawal, 0 = no more transactions), a six-digit date (in the form DDMMYY), and the amount of transaction.

Solution.

```
INTEGER CODE, DATE
REAL BALNCE, AMOUNT

READ *, BALNCE
PRINT *, ' ', BALNCE

10 READ *, CODE, DATE, AMOUNT

IF (CODE .EQ. 0) THEN
STOP
```

```
ELSE IF (CODE .EQ. 1) THEN
BALNCE = BALNCE + AMOUNT
PRINT *, 'DEP', DATE, AMOUNT, BALNCE

ELSE IF (CODE .EQ. 2) THEN
BALNCE= BALNCE - AMOUNT
PRINT *, 'WDR', DATE, AMOUNT, BALNCE

ELSE
PRINT *, 'INALID CODE READ'
ENDIF

GO TO 10

END
```

Q. 29. Write a FORTRAN program that goes on reading values for an integer variable N until the value read is zero or negative. For each positive value of N read, the program tests whether N is a prime number or not. Also it should print appropriate message. (Dec. 2001)

Solution.

```
INTEGER N, I, J, K, L
WRITE (*, 10)
10    FORMAT(1X, 'ENTER VALUE OF N = ',\)
READ (*, 11) N
11    FORMAT(I3)
100   IF (N .LE. 0) THEN GOTO 200
K = 0
I = 2
20    IF (I .GT. N/2) THEN GOTO 40
IF (MOD (N, I) .NE. 0) THEN
WRITE (*, 30) N
30    FORMAT(1X, 'I3 IS NOT A PRIME NUMBER')
GOTO 150
ENDIF
I = I + 1
GOTO 20
40    WRITE (*, 50) N
50    FORMAT(1X, 'I3 IS A PRIME NUMBER')
150   WRITE (*, 55)
55    FORMAT(1X, 'ENTER VALUE OF N = ',\)
READ (*, 60) N
```

```
60    FORMAT(I3)
GOTO 100
200    STOP
END
```

Q. 30. Write a FORTRAN program which finds all prime numbers between 1000 and 9999. (June 2000)

Solution.

```
INTEGER N, K, M
5    DO 20 N = 1000, 9999
K = 2
M = SQRT (N)
10   IF (MOD (N, K) .EQ. 0) GOTO 5
K = K + 1
IF (K .LE. M) THEN GOTO 10
WRITE (*, 15) N
15    FORMAT(1X, I5, 'IS A PRIME NUMBER')
20    CONTINUE
STOP
END
```

Q. 31. Write a FORTRAN program which finds all three digit prime numbers. (Dec. 99)

Solution.

```
INTEGER N, K, M
10    DO 40 N = 100, 999
K = 2
M = SQRT (N)
20   IF (MOD (N, K) .EQ. 0) GOTO 10
K = K + 1
IF (K .LE. M) THEN GOTO 20
WRITE (*, 30) N
30    FORMAT(1X, I5, 'IS A PRIME NUMBER')
40    CONTINUE
STOP
END
```

Q. 32. Write a FORTRAN subroutine that computes the sum of square of the minimum and cube of the maximum of an array of N natural numbers, and prints out the value with suitable message. (June 2000)

Solution.

```
SUBROUTINE TOT (A, N, SUM)
REAL A(N),SUM, MAX, MIN
INTEGER I, N
SUM = 0.0
MAX = A(1)
MM = A(1)
DO 10 I = 2, N
IF (A(I) .GT. MAX) MAX = A(I)
IF (A(I) .LT. MIN) MIN = A(I)
10    CONTINUE
SUM = SUM + MIN * * 2 + MAX * * 3
WRITE (*, 15) SUM
15    FORMAT(1X, 'THE SUM OF SQUARE OF MIN AND CUBE OF MAX=', F5.2)
STOP
END
```