Sample Paper – 2007
Mathematics
Class - XII

Time Allowed : 3 hrs **CLASS –XII-Mathematics** ***Max Marks : 100***

General Instructions :

(i) The question paper consists of three sections A, B , C .Section A is compulsory for all students. In addition to section A, every student has to attempt either section B or Section C.

(ii) For section A
Questions number 1 to 8 are of 3 marks each.
Questions number 9 to 15 are of 4 marks each.
Questions number 16 to18 are of 6 marks each.

(iii) For Section B/ Section C
Questions number 19 to 22 are of 3 marks each.
Questions number 23 to 25 are of 4 marks each.
Questions number 26 is of 6 marks.

(iv) All questions are compulsory.

(v) Internal choice have been provided in some questions. You have to attempt only one of the choices in such questions.

(vi) Use of calculator is not permitted. However, you may ask for logarithmic and & statistical tables , if required.

SECTION-A

1. If $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$, show that (A - A^T) is a skew-symmetric matrix, where A^T is the transpose of matrix A.

2.Prove that : $\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \gamma+\alpha & \alpha+\beta\end{vmatrix}=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$.

3. A pair of dice is thrown 7 times.If getting a total of 7 is considered a success, what is the probability of atmost 6 successes ?

4. Two cards are drawn one by one without replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

5. Evaluate : $\int\frac{dx}{1+\cot x}$

6.Evaluate : $\int\frac{1}{\sqrt{(x-1)(x-2)}}dx$

7.Solve the differential equation : $(1+e^{2x})dy + e^{x}(1 + y^{2})dx = 0$, Given that y = 1, when x = 0.

8. Solve the initial value problem $(x^2+1)\frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1)\cos x$, y(0) = 0.

OR

Solve $\frac{dy}{dx} = 3y\cot x + \sin 2x$

9.Examine the validity of the following argument :

$S_1 : p \vee (q \vee r)$

$S_2 :\sim r$

$S : p \vee q$

OR

Construct a combinotrial circuit for the following Boolean expressions :

(i) $[(x_1.x_2) + (x_1.x'_2)] .x_2$

(ii) $[x_1.(x'_2+x_3)] +x_2$

10. Show that : $\lim_{x\to 0}\frac{a^x + b^x + c^x - 3}{x} = \log(abc)$.

11. Prove that : $\frac{d}{dx}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{1}{2}$

12. Verify Rolle's theorem for the function $f(x) = x^3 + 3x^2 - 24x - 80$ in the interval [4,5].

13. Find the derivative of **log tanx** by Ab-initio method.

14. Evaluate : $\int \frac{dx}{\cos\alpha + \cos x}$

15. Prove that : $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

OR

$\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a-x)dx$

16. Draw a rough sketch of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$. Using method of integration, find the area of this enclosed region.

17. A window is in the form of rectangle above which there is semi-circle. If the perimeter of the window is 'p'cm,show that the window will allow the maximum possible light only when the radius of the semi-circle is $\frac{p}{\pi+4}cm.$

OR

An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width.

18. Solve the following system of equations , using matrices :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

SECTION-B

19. If $\vec{a}+\vec{b}+\vec{c}=0$ where $\vec{a},\vec{b},\vec{c}$ are unit vectors.Find $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

20 Forces $2\hat{i}+7\hat{j}$, $2\hat{i}-5\hat{j}+6\hat{k}$, $-\hat{i}+2\hat{j}-\hat{k}$ act at a point P whose position vector is $4\hat{i}-3\hat{j}-2\hat{k}$ .Find the vector moment of the resultant of three forcesacting at point P about the point Q whose position vector is $6\hat{i}+\hat{j}-3\hat{k}$.

21. Find the perpendicular distance of the point (1,0,0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$.

22. Prove that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$,and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ are coplanar.Also,find the plane containing these lines.

23. Find the radius of the circular section of the sphere $x^2 + y^2 + z^2 = 49$ by the plane $2x + 3y – z - 5\sqrt{14} = 0$.

24.A man carries a bundle at one end of the a stick which is placed over his shoulder . If ‘W’ be the weight of the bundle and ‘a’ and ‘b’ be the respective distances of the bundle and his hand from the shoulder. Show that the pressure on the shoulder is $W\left(1+\frac{a}{b}\right)$. If the distance between his shoulder and hand be changed ,how does the pressure on his shoulder changes .

OR

State Lami’s theorem.Using it, find the ratio of the magnitude of three forces P,Q,R which are in equilibrium and are acting at a point in the same plane such that P makes with Q and R the angles of 120^0 and 90^0 respectively.

25. Forces of magnitudes 1,2,3,4,5 and 6N act along the sides AB, BC,CD , DE, EF and FA of regular hexagon. Find the moment of each forces about A.

26.A particle is projected so as to have a a range R on the horizontal plane through the point of projection. If α and β are the possible angles of projection and t_1 and t_2 the corresponding times of flights, show that $\frac{t_1^{\,2}-t_2^{\,2}}{t_1^{\,2}+t_2^{\,2}}=\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$

OR

A particle is projected with a velocity 'u' so as just to pass the heighest possible post at a horizontal distance 'd' from the point of projection O. Prove that the greatest height above O attained by particle in its flight is $\frac{u^6}{2g(u^4+g^2d^2)}$.

SECTION-C

19. A speaks the truth 8 times out of 10 times . A die is tossed. He reports that it was 5.What is the probabilityn that it was actually 5.

20. A coin is tossed 4 times.Find the mean and variance of the probability distribution of the number of heads.

OR

For a Poisson distribution, it is given that P(x =1) = P(x = 2).Find the value of the mean of the distribution. Hence find P(x = 0) and P(x = 4). [Use e^{-2} =0.13534].

21. If the banker's gain on a bill be 1/9th of the banker's discount, the rate of interest having 10% p.a., find the expired period of the bill

22. A bill of Rs. 5,300, drawn on 16th January, 2003 fro 8 months was discounted on 12th February, 2003 at 10% p.a.,Find the banker's gain and discounted value of the bill.

23. In a business partnership, **A** invests half of the capital for the half of the period.**B** invests one-third of the capital for one-third of the period, and **C** invest rest of the capital for the whole period. Find the share of each in the profit of Rs. 1,90,000.

24.'A' plans to buy a new flat after 5 years,which will cost him Rs. 5,52,000. How much money should he deposite annually to accumulate this amount, If he get interest 5% p.a. compounded annually ?

25. The cost function of a firm is given by :

$C(x) = 300x - 10x^2 + x^3/3$, where x stands for output , calculate:

(i) The output at which the marginal cost is minimum.

(ii) The output at which the average cost is equal to the marginal cost.

26. Solve the following linear programming problem graphically.

Minimize Z = 3x + 5y subject to constraints :

$X + y \geq 2$; $x + 3y \geq 3$; $x , y \geq 0$

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